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by

Stefan Baumgärtner, Moritz A. Drupp,  
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# Subsistence and substitutability in consumer preferences

STEFAN BAUMGÄRTNER<sup>a,\*</sup>, MORITZ A. DRUPP<sup>b,c</sup> and MARTIN F. QUAAS<sup>c</sup>

<sup>a</sup> Department of Sustainability Science and Department of Economics,  
Leuphana University of Lüneburg, Germany

<sup>b</sup> Department of Geography and Environment,  
London School of Economics and Political Science, UK

<sup>c</sup> Department of Economics, University of Kiel, Germany

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**Abstract:** We propose a formal description of individual preferences that captures a subsistence requirement in consumption in an otherwise standard constant-elasticity-of-substitution (CES) utility specification. We study how substitutability between the subsistence good and another good depends on the subsistence requirement and the level of consumption of the two goods. We find that the Hicksian elasticity of substitution is zero below the subsistence consumption level, and approaches the standard non-subsistence CES value as consumption of the subsistence good goes to infinity. Above the subsistence threshold, it strictly monotonically increases with income. Whether the two goods are market substitutes or complements depends on, besides the CES-substitutability parameter, the level of income and the subsistence requirement. Our result that with a subsistence requirement substitutability between different consumption goods is non-constant but increases with individual income has important implications for growth, development and environmental policy.

**JEL-Classification:** D11, I31, O12, Q01

**Keywords:** subsistence in consumption, substitutability, Hicksian elasticity of substitution, Stone-Geary function

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\*Corresponding author: Sustainability Economics Group, Leuphana University of Lüneburg, P.O. Box 2440, D-21314 Lüneburg, Germany. Phone: +49.4131.677-2600, fax: +49.4131.677-1381, email: baumgaertner@uni.leuphana.de.

# 1 Introduction

We develop a general and formal conceptual framework to examine substitutability between two goods in the presence of a subsistence requirement in the consumption of one of these goods.

The economic study of subsistence requirements in consumption dates back to Klein and Rubin (1947/48), Samuelson (1947/48), Geary (1950) and Stone (1954). More recently, subsistence requirements have been shown to be relevant, *inter alia*, in the growth, development and environmental economics literature (e.g. Garner 2010, Heal 2009, King and Rebelo 1993, Kraay and Raddatz 2007, Matsuo and Tomoda 2012, Pezzey and Anderies 2003, Ravn et al. 2008, Steger 2000, Strulik 2010). Despite extensive discussion, there is no consensus yet on the appropriate definition of subsistence (Alkire 2002, Heal 2009, Max-Neef et al. 1991, Rauschmayer et al. 2011, Sharif 1986).<sup>1</sup>

Here, we understand subsistence to capture more than mere survival, but to encompass a homogeneous composite good to which an individual attaches absolute priority before considering trade-offs with other goods. This certainly includes the consumption of a certain amount of food, water and air, but may also include immaterial components.

While substitutability between consumption goods in the presence of a subsistence requirement has for long time not been a focus of study, it is remarkable that Hicks and Allen (1934b: 199) already had this case in mind when first formulating the elasticity of substitution in consumption. Only recently, Heal (2009) proposed to examine substitutability between an environmental and a produced good in the presence of a subsistence requirement in terms of the environmental good by extending a constant-elasticity-of-substitution (CES) utility function through including a survival threshold. Without providing a formal examination, Heal (2009: 279) conjectures that “the elasticity of substitution is not constant but depends on and increases with welfare levels.”

In this paper, we generalize and formalize Heal’s (2009) proposal by incorporating a subsistence requirement in an otherwise standard CES utility function. We indeed

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<sup>1</sup>For instance, while Sharif (1986) argues that subsistence includes both physical and mental health, Heal (2009: 279) conceptualizes a survival threshold in terms of “water, air, and basic foodstuffs”.

find that the Hicksian elasticity of substitution is non-constant and, above the subsistence threshold, strictly monotonically increases with income. However, whether the two goods are market substitutes does not only depend on the Hicksian elasticity of substitution, but also on the level of income and the subsistence requirement. We further find that the degree of market substitutability increases with an individual's income.

## 2 Model and definitions

There are two composite goods,  $S$  (say, food) and  $X$  (all the rest), with a subsistence requirement  $\bar{S}$  with respect to the first good. The consumer's preferences are represented by a utility function

$$U(S, X) = \begin{cases} U_l(S) & \text{for } S \leq \bar{S} \\ U_h(S, X) & \text{else} \end{cases} \quad (1)$$

where  $U_l(\cdot)$  is a strictly monotonically increasing function of  $S$ , and  $U_h(\cdot, \cdot)$  is a twice continuously differentiable function which is strictly monotonic in both arguments and strictly quasi-concave. Furthermore, the individual always prefers to be in the domain where the subsistence requirement is satisfied, i.e.

$$\inf_{S > \bar{S}, X \geq 0} U_h(S, X) > \sup_{0 \leq S \leq \bar{S}} U_l(S) . \quad (2)$$

Utility function (1) represents the idea of subsistence requirements in consumption: for  $S \leq \bar{S}$  the individual is not willing to consider trade-offs between the subsistence good  $S$  and the other good  $X$ , but lexicographically prefers more of  $S$ . Only if subsistence consumption is satisfied, i.e. for  $S > \bar{S}$ , is she willing to consider trade-offs between  $S$  (insofar it exceeds  $\bar{S}$ ) and  $X$ .

As an interesting and handy specification of  $U_h(\cdot, \cdot)$  we suggest, following Heal's (2009) idea, a generalized modification of the Stone-Geary function (Geary 1950, Stone 1954)<sup>2</sup> on the one hand, and the CES-function (Solow 1956, Arrow et al. 1961) on the

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<sup>2</sup>The Stone-Geary function has originally been proposed by Klein and Rubin (1947/48) and Samuelson (1947/48).

other, which contains these two functions as special cases:<sup>3</sup>

$$U_h(S, X) = \left[ \alpha (S - \bar{S})^\theta + (1 - \alpha) X^\theta \right]^{1/\theta} \quad \text{with} \quad -\infty < \theta \leq +1. \quad (3)$$

For  $\bar{S} = 0$ , i.e. without subsistence requirement, this function reduces to the usual CES utility function which contains as special case perfect substitutes ( $\theta = +1$ ), Cobb-Douglas ( $\theta = 0$ ) and perfect complements ( $\theta = -\infty$ ).

As a measure of substitutability between the two goods, we use the ‘direct’ elasticity of substitution introduced by Hicks (1932[1963]), Robinson (1933) and Hicks and Allen (1934).<sup>4</sup> This is the most basic measure of substitutability, and a rather general one. In contrast to more sophisticated measures – such as e.g. gross, net or Morishima substitutability<sup>5</sup> – it does not rely on any further assumptions on individual behavior or the institutional context (e.g. utility-maximizing and price-taking behavior on competitive markets), but it characterizes individual preferences and preferences only.

### Definition 1

The *elasticity of substitution*  $\sigma$  is the elasticity of the ratio of the amounts of the two goods in a given allocation  $(S, X)$  with respect to the marginal rate of substitution in that allocation:

$$\sigma(S, X) := \frac{MRS}{X/S} \frac{d(X/S)}{dMRS}, \quad (4)$$

where

$$MRS := - \left. \frac{dX}{dS} \right|_{U(S,X)=\text{const.}} = \frac{\frac{\partial U(S, X)}{\partial S}}{\frac{\partial U(S, X)}{\partial X}}. \quad (5)$$

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<sup>3</sup>Although the CES function has originally been proposed as a production function, it is widely used as a utility function since Armington (1969). Note also that specification (3) is itself a special case of the ‘affinely homothetic’ S-branch utility tree (Brown and Heien 1972, Blackorby et al. 1978).

<sup>4</sup>Hicks (1932[1963]) and Robinson (1933) independently introduced the elasticity of substitution between two inputs to production, which has then been adapted to consumption goods by Hicks and Allen (1934a,b). For a generalization to more than two goods, see Blackorby and Russel (1989).

<sup>5</sup>See Bertoletti (2005), Frondel (2011) and Stern (2011) for useful overviews.

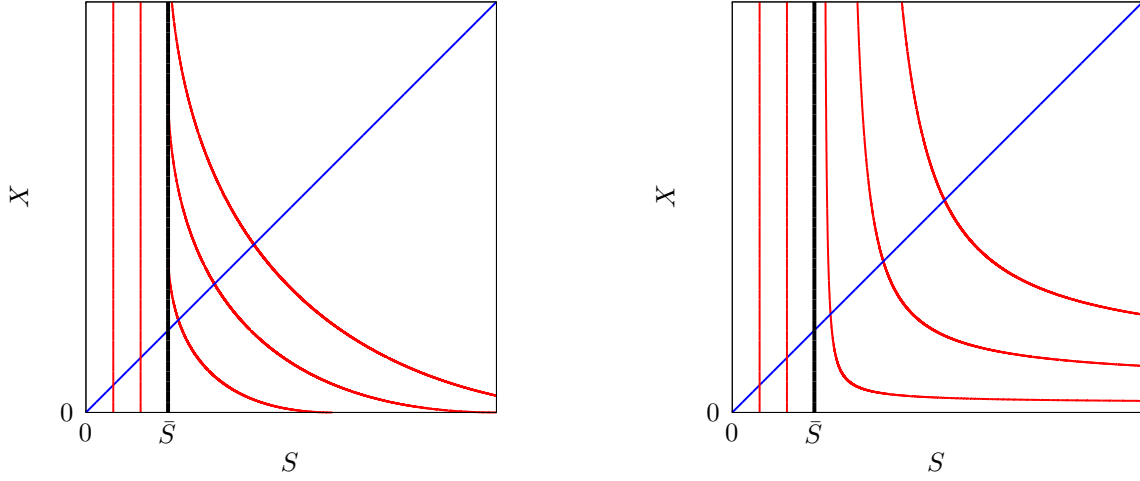


Figure 1: Sets of indifference curves for  $\theta = 0.5$  (left), and  $\theta = -0.5$  (right). The blue 45-degree line highlights that preferences are non-homothetic.

Note that for the utility function (1) considered here, the marginal rate of substitution (5) is only defined for the domain  $S > \bar{S}$ , i.e. for  $U_h(\cdot, \cdot)$ . Since the individual is not willing to trade off  $S$  for  $X$  in the domain  $S \leq \bar{S}$ , we plausibly extend Definition 1 by defining the elasticity of substitution to be equal to zero for  $U_h(\cdot)$ .

As is evident from Definition 1, the elasticity of substitution measures substitutability between two goods along an indifference curve. Previous studies (e.g. Brown and Heien 1972, Beckman and Smith 1993) that have analyzed the elasticity of substitution for affinely homothetic utility functions have not defined the elasticity of substitution with respect to the origin (here:  $S = 0, X = 0$ ), but instead with respect to the subsistence requirement bundle (here:  $S = \bar{S}, X = 0$ ), thus confining themselves to a standard CES setting. However, it is reasonable and straightforward, following a recommendation by Hicks (1932 [1936: 296]), to define the elasticity of substitution also in the case of this non-homothetic function with respect to the true origin ( $S = 0, X = 0$ ).

### 3 Results

For  $\bar{S} = 0$ , i.e. without subsistence requirement, utility function (1) with specification (3) reduces to the usual CES utility function with

$$\sigma(S, X) = \frac{1}{1 - \theta} = \text{const.} \quad (6)$$

The constant parameter  $\theta$  completely determines the elasticity of substitution, with the special cases of perfect substitutes ( $\theta = +1$ ), Cobb-Douglas ( $\theta = 0$ ) and perfect complements ( $\theta = -\infty$ ). If, in contrast, there is a subsistence requirement, one obtains:

**Proposition 1**

For  $\bar{S} > 0$ , the elasticity of substitution of utility function (1) with specification (3) is given by

$$\sigma(S, X) = \begin{cases} 0 & \text{for } S \leq \bar{S} \\ \frac{1}{1 - \theta} \left[ 1 - \frac{(1 - \alpha) \frac{\bar{S}}{S}}{\alpha \left[ \frac{S - \bar{S}}{X} \right]^\theta + (1 - \alpha)} \right] & \text{else.} \end{cases} \quad (7)$$

*Proof.* See Appendix A.1. □

By Definition 1, the elasticity of substitution is zero as long as each further unit of  $S$  is necessary to meet the subsistence requirement  $\bar{S}$ . More interesting are the properties of the elasticity of substitution in the domain where subsistence consumption is satisfied. Here, the value of  $\sigma$  is determined by the parameter value of  $\theta$  and the amounts consumed of both goods in a given allocation  $(S, X)$ , as well as the subsistence requirement  $\bar{S}$ .

**Proposition 2**

For  $S > \bar{S}$  and  $X > 0$ , the elasticity of substitution  $\sigma(S, X)$  (Equation 7) has the following properties:



$$0 < \sigma(S, X) < \frac{1}{1-\theta} \quad \text{for all } (S > \bar{S}, X) \quad (8)$$

$$\frac{d\sigma(S, X)}{dS} > 0 \quad \text{for } \theta \geq 0 \quad (9)$$

$$\sigma(S, X) = \begin{cases} 0 \\ \alpha \\ \frac{1}{1-\theta} \end{cases} \quad \text{for } S \rightarrow \bar{S} \quad \text{and } \theta \begin{matrix} \geq \\ = \\ \leq \end{matrix} 0 \quad (10)$$

$$\sigma(S, X) \rightarrow \frac{1}{1-\theta} \quad \text{for } S \rightarrow \infty \quad (11)$$

$$\frac{d\sigma(S, X)}{d\bar{S}} < 0 \quad \text{for } \theta \geq 0 \quad (12)$$

$$\sigma(S, X) \rightarrow \frac{1}{1-\theta} \quad \text{for } \bar{S} \rightarrow 0 \quad (13)$$

$$\sigma(S, X) \rightarrow \infty \quad \text{for } \theta \rightarrow 1 \quad (14)$$

*Proof.* See Appendix A.2. □

Property (8) states that the elasticity of substitution can take on any value between zero and the non-subsistence CES value  $\sigma = 1/(1-\theta)$ . In particular, it is always strictly smaller than the non-subsistence CES value. That is, the subsistence requirement shifts the relationship between the two goods towards complementarity.

Property (9) states that for non-negative values of  $\theta$  the elasticity of substitution in any given allocation is the greater, the higher the consumption of good  $S$ . Property (10) states that as the consumption of good  $S$  approaches the subsistence requirement from above, the elasticity of substitution approaches zero for  $\theta > 0$ , is equal to the share  $\alpha$  of good  $S$  for  $\theta = 0$  and approaches the standard CES result for  $\theta < 0$ . Property (11) states that as consumption of the subsistence good goes to infinity, the elasticity of substitution approaches the non-subsistence CES value  $\sigma = 1/(1-\theta)$ .

Property (12) states that for non-negative values of  $\theta$  the elasticity of substitution is the smaller, the higher the subsistence requirement. Property (13) states that in the limit case where no subsistence requirement exists the elasticity of substitution approaches

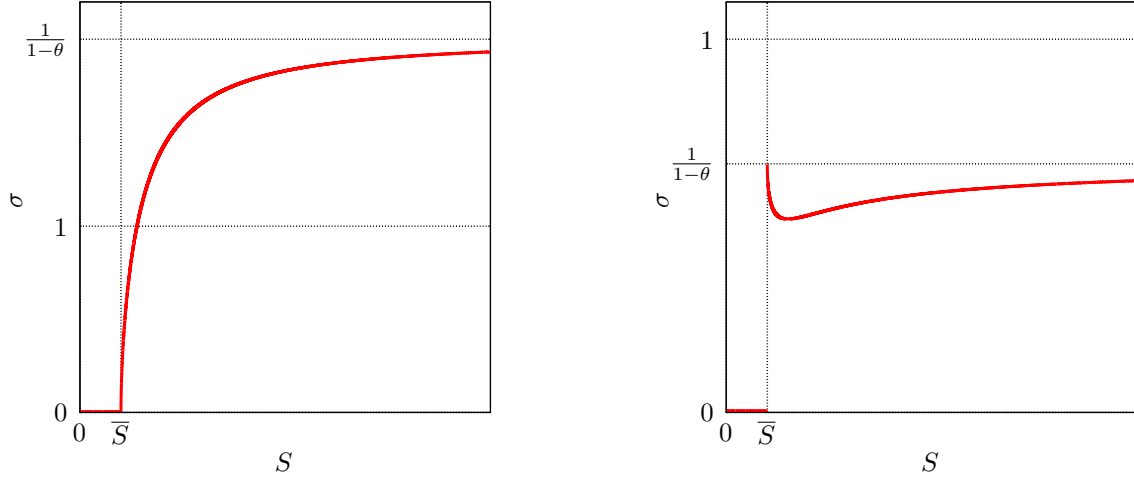


Figure 2: Elasticity of substitution  $\sigma(S, X)$  (Equation 7) as a function of consumption of the subsistence good  $S$  for  $\theta = 0.5$  (left) and  $\theta = -0.5$  (right).

the standard non-subsistence CES value  $\sigma = 1/(1 - \theta)$ . Property (14) states that when  $\theta$  approaches unity, i.e.  $S$  and  $X$  are perfect substitutes, the elasticity of substitution becomes infinite, irrespectively of the level of consumption of  $S$  or  $X$ .

Some of these properties are illustrated in Figure 2. It shows the elasticity of substitution  $\sigma(S, X)$  (Equation 7) as a function of the consumption of the subsistence good  $S$  for different signs of  $\theta$ . It is particularly noteworthy that for  $\theta > 0$  (cf. Figure 2 left), a subsistence good that is considered a substitute to the other consumption good when available in large quantity (high  $S$ ), can change to become a complement as its availability is reduced (low  $S$ ). In contrast, for  $\theta < 0$  (cf. Figure 2 right) the subsistence good is a complement to the other consumption good at all levels of consumption  $S$ . This is a direct implication of the subsistence requirement, which shifts the relationship between goods towards complementarity.

Having analyzed the elasticity of substitution in pure preference space, we now add institutional context to the analysis by invoking a market setting, i.e. given income  $m$  and prices  $p_S$  and  $p_X$ . The utility-maximizing allocation of the two goods under the budget constraint, assuming price-taking behavior of the consumer, is then given

by the respective Marshallian demand functions  $S^*(m, p_S, p_X)$  and  $X^*(m, p_S, p_X)$ . We denote by  $\sigma^* := \sigma(X^*, S^*)$  the elasticity of substitution in the individual's utility-optimal allocation.

**Proposition 3**

The elasticity of substitution in the optimal allocation,  $\sigma^*$ , has the following properties:

$$\sigma^* = 0 \quad \text{for} \quad m \leq p_S \bar{S} \quad (15)$$

For  $m > p_S \bar{S}$ , the following holds:

$$0 < \sigma^* < \frac{1}{1-\theta} \quad (16)$$

$$\frac{d\sigma^*}{dm} > 0 \quad (17)$$

$$\sigma^* \rightarrow \frac{\alpha}{\alpha + (1-\alpha) \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{\theta}{\theta-1}}} \frac{1}{1-\theta} < \frac{1}{1-\theta} \quad \text{for} \quad m \rightarrow p_S \bar{S} \quad (18)$$

$$\sigma^* \rightarrow \frac{1}{1-\theta} \quad \text{for} \quad m \rightarrow \infty \quad (19)$$

$$\frac{d\sigma^*}{d\bar{S}} < 0 \quad (20)$$

For  $\theta > 0$ , there is a threshold value of income

$$m^P = p_S \bar{S} \frac{1-\theta}{\theta} \left( \frac{1-\alpha}{\alpha} \right)^{\frac{1}{1-\theta}} \left( \frac{p_S}{p_X} \right)^{\frac{\theta}{1-\theta}} \quad (21)$$

such that

$$\sigma^* \begin{matrix} \leq \\ \geq \end{matrix} 1 \quad \text{for} \quad m \begin{matrix} \leq \\ \geq \end{matrix} m^P. \quad (22)$$

*Proof.* See Appendix A.3. □

Property (15) follows directly from Definition 1 and states that the elasticity of substitution between the two goods in the optimal allocation is zero as long as income is not high enough to afford a consumption level satisfying the subsistence requirement.

Properties (16) – (20) correspond to the statements of Proposition 2 about  $\sigma(S, X)$  but are considerably stronger, as they now hold for all values of the CES-parameter  $\theta$ . In

particular, in the domain where the subsistence requirement is satisfied ( $m > p_S \bar{S}$ ), the elasticity of substitution in the optimal allocation increases strictly monotonically with income (Property 17) and with the level of the subsistence requirement (Property 20).

Again, the elasticity of substitution approaches the standard non-subsistence CES value as income goes to infinity (Property 19). This finding implies that with a subsistence requirement the non-subsistence CES value only holds in a world of plenty where income (to purchase the subsistence good) is available in an infinite amount. The argumentum a contrario is that in the case of finite income, the elasticity of substitution between the two goods is strictly lower in the presence of a subsistence requirement than in the standard CES case (Property 16). So, the substitutability relationship between the two goods is shifted towards complementarity.

Property (22) states that, for  $\theta > 0$ , there is a threshold level of income  $m^P$  (Equation 21), depending on all model parameters, so that the two goods are substitutes (complements) in preferences, i.e. in terms of the Hicksian elasticity of substitution, for incomes above (below)  $m^P$ . This threshold income level increases with the subsistence requirement  $\bar{S}$  and the price of the subsistence good  $p_S$ , it decreases with the price of the other good  $p_X$ . Its dependence on the CES-parameter is ambiguous.

Finally, we analyze whether the two goods are market substitutes (complements) in the sense that if the price of one good increases the demand for the other goods increases as well (decreases):

#### Proposition 4

Assuming that income is higher than necessary to meet the subsistence requirement,  $m > p_S \bar{S}$ , the cross-price effect on the Marshallian demand of the two goods is as follows:

$$\frac{d S^*}{d p_X} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad \text{for } \theta \begin{matrix} \geq \\ \leq \end{matrix} 0, \quad (23)$$

$$\frac{d X^*}{d p_S} < 0 \quad \text{for } \theta \leq 0. \quad (24)$$

For  $\theta > 0$ , there exists a threshold value of income

$$m^M = \frac{p_S \bar{S}}{\theta} + m^P \quad (25)$$

such that

$$\frac{dX^*}{dp_S} \begin{matrix} \leq \\ \geq \end{matrix} 0 \quad \text{for} \quad m \begin{matrix} \leq \\ \geq \end{matrix} m^M. \quad (26)$$

*Proof.* See Appendix A.4. □

Property (23) states that whether good  $X$  is a market-substitute for good  $S$  only depends on the CES parameter  $\theta$ , which is proportional to the Hicksian elasticity of substitution in the standard non-subsistence CES case.

The effect of an increase of the price of  $S$  on the Marshallian demand for  $X$  is more nuanced, and in particular depends on, and monotonically increases with, income: Property (24) states that  $X$  is a market-complement for  $S$  if  $\theta \leq 0$ . In the case of  $\theta > 0$ , whether the subsistence good  $S$  is a market-substitute for  $X$  depends on all model parameters,<sup>6</sup> and in particular on income: as stated by Result (26), the subsistence good  $S$  is only a substitute for  $X$  for a sufficiently high level of income, and a complement otherwise.

Interestingly, the threshold value of income  $m^M$  (Equation 25) that determines market substitutability is a simple additive extension to the income threshold that determines the Hicksian substitutability (Equation 21) with the income necessary to satisfy subsistence needs  $p_S \bar{S}$  weighted by the CES-substitutability parameter  $\theta$ . As  $m^P > 0$  and  $0 < \theta < 1$ , one has that  $m^M > p_S \bar{S}$ .  $m^M$  increases with the subsistence requirement  $\bar{S}$  and the price of the subsistence good  $p_S$ , it decreases with the price of the other good  $p_X$ . Its dependence on the CES-parameter is ambiguous. The intuition behind Result (25) is that as the price of the subsistence good increases, it requires higher income to meet the subsistence requirement, thus also shifting the market-substitutability relationship towards complementarity (see Figure 3).

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<sup>6</sup>Except for the extreme case of  $\theta \rightarrow 1$ , where the two are perfect market-substitutes irrespective of any other parameter values (cf. Equation A.45 in Appendix A.4.)

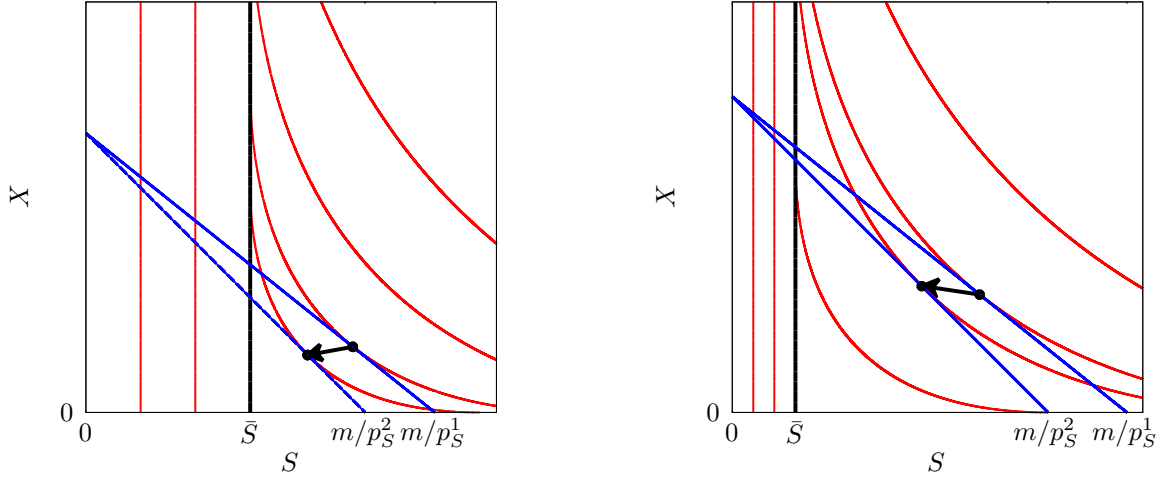


Figure 3: Optimal consumption of  $S$  and  $X$  for low income (left,  $m = 1.7$ ) and high income (right,  $m = 5$ ) in the case  $\theta = 0.5$  and  $\bar{S} = 1$ . Indifference curves are depicted in red, budget constraints in blue for  $p_X = 1$  and two values of  $p_S$ ,  $p_S^1 = 0.8$  and  $p_S^2 = 1$ . At low income (left), an increase of  $p_S$  (from  $p_S^1$  to  $p_S^2$ ) decreases demand for  $X$ ; at high income (right), the same increase of  $p_S$  increases demand for  $X$ .

## 4 Discussion and conclusions

We have proposed a formal description of individual preferences that captures a subsistence requirement in consumption in an otherwise standard constant-elasticity-of-substitution (CES) utility specification. We have studied how substitutability between the subsistence good and another good depends on the subsistence requirement and the level of consumption of the two goods.

We find that (i) a subsistence requirement shifts the substitutability relationship between goods towards complementarity; (ii) the Hicksian elasticity of substitution is equivalent to the standard non-subsistence CES value only if the subsistence good or income is available in infinite amount; (iii) above the subsistence threshold, the elasticity of substitution strictly monotonically increases with income; (iv) whether the two goods are market substitutes or complements depends on, besides the CES-substitutability

parameter, the level of income and the subsistence requirement.

While our analysis is set in the framework of a specific and relevant functional form – a simple subsistence requirement extension of an otherwise standard CES utility function – our main results remain valid under more general preference specifications.

Our main result that with a subsistence requirement, substitutability between different consumption goods is non-constant but increases with individual income, has important implications, in particular for growth, development and environmental policy. These need to be explored by further research, and we can think of several fruitful areas for such research:

First, the connections between income distribution and Pareto-efficiency of market allocations (i.e. the first and second welfare theorems) need to be reassessed for preferences characterized by a subsistence requirement. It would be particularly interesting to examine a two-household model in which one household may have an insufficient budget to meet the subsistence requirement. This may lead *inter alia* to poverty traps in subsistence economies.

Second, since substitutability between manufactured consumption goods and environmental services is a key issue in the appraisal of climate policies (Heal 2009, Sterner and Persson 2008), the application of this subsistence-substitutability model will have directly relevant implications for the optimal management of climate change.

Third, and more generally, in the discussion of sustainable development, the distinction between weak and strong sustainability is important and vindicated on the grounds of different degrees of substitutability between human-made and natural capital and the respective services (Neumayer, 2010; Trager, 2011). With our preference-model in a general-equilibrium setting, the distinction becomes endogenous, as the elasticity of substitution depends on income.

Fourth, the standard result of growth and resource economics by Solow (1974) that a constant consumption path forever is feasible even if production essentially depends on a non-renewable resource needs to be qualified: if the Solowesque constant consumption level is below the subsistence threshold, one would not like to think of that solution as “sustainable”.

## Acknowledgments

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## Appendix

### A.1 Proof of Proposition 1

With utility function (3), the marginal rate of substitution for  $S > \bar{S}$  (Equation 5) is

$$MRS = \frac{\frac{\partial U_h(S, X)}{\partial S}}{\frac{\partial U_h(S, X)}{\partial X}} = \frac{\alpha}{1 - \alpha} \left[ \frac{S - \bar{S}}{X} \right]^{\theta - 1} = \frac{\alpha}{1 - \alpha} \left[ \frac{S}{X} \left( 1 - \frac{\bar{S}}{S} \right) \right]^{\theta - 1}, \quad (\text{A.1})$$

so that

$$\frac{MRS}{(X/S)} = \frac{\alpha}{1 - \alpha} \left( \frac{S}{X} \right)^{\theta} \left( 1 - \frac{\bar{S}}{S} \right)^{\theta - 1} \quad (\text{A.2})$$

and

$$\frac{d MRS}{d(X/S)} = \frac{\alpha(\theta - 1)}{1 - \alpha} \left[ \frac{S}{X} \left( 1 - \frac{\bar{S}}{S} \right) \right]^{\theta - 2} \left\{ \frac{d(S/X)}{d(X/S)} \left( 1 - \frac{\bar{S}}{S} \right) + \frac{S}{X} \frac{d \left( 1 - \frac{\bar{S}}{S} \right)}{d(X/S)} \right\} \quad (\text{A.3})$$

$$= \frac{\alpha(\theta - 1)}{1 - \alpha} \left[ \frac{S}{X} \left( 1 - \frac{\bar{S}}{S} \right) \right]^{\theta - 2} \left\{ - \left( \frac{X}{S} \right)^{-2} \left( 1 - \frac{\bar{S}}{S} \right) + \frac{S}{X} \frac{d \left( 1 - \frac{\bar{S}}{S} \right)}{d(X/S)} \right\} \quad (\text{A.4})$$

With (A.2) and (A.4), the elasticity of substitution (4) becomes

$$\sigma(S, X) = \frac{MRS}{X/S} \frac{d(X/S)}{dMRS} = \frac{MRS}{X/S} \left( \frac{dMRS}{d(X/S)} \right)^{-1} \quad (\text{A.5})$$

$$= \left( \frac{S}{X} \right)^\theta \left( 1 - \frac{\bar{S}}{S} \right)^{\theta-1} \frac{1}{\theta-1} \left[ \frac{S}{X} \left( 1 - \frac{\bar{S}}{S} \right) \right]^{2-\theta} \times \\ \times \left\{ - \left( \frac{X}{S} \right)^{-2} \left( 1 - \frac{\bar{S}}{S} \right) + \frac{d \left( 1 - \frac{\bar{S}}{S} \right)}{d(X/S)} \frac{S}{X} \right\}^{-1} \quad (\text{A.6})$$

$$= \frac{1}{1-\theta} \left\{ 1 - \frac{\frac{X}{\bar{S}}}{\left( 1 - \frac{\bar{S}}{S} \right)} \frac{d \left( 1 - \frac{\bar{S}}{S} \right)}{d(X/S)} \right\}^{-1}. \quad (\text{A.7})$$

To calculate the remaining derivative, we transform the problem from the standard variables  $(S, X)$  into the following variables  $(w, v)$ :

$$w := \frac{X}{S} \quad (\text{A.8})$$

$$v := \alpha (S - \bar{S})^\theta + (1 - \alpha) X^\theta, \quad (\text{A.9})$$

where  $w$  is the ratio of the two consumption goods and  $v$  is a monotonic transformation of the utility function, so that  $v = \text{constant}$  is equivalent to  $U(S, X) = \text{constant}$ . Derivatives under the constraint  $U(S, X) = \text{const.}$ , i.e. along an indifference curve, are now taken along  $v = \text{const.}$ , or  $dv = 0$ .

From (A.9), using (A.8), we have

$$\left( 1 - \frac{\bar{S}}{S} \right) = \left[ \frac{v}{\alpha S^\theta} - \frac{1-\alpha}{\alpha} w^\theta \right]^{\frac{1}{\theta}}, \quad (\text{A.10})$$

such that

$$\frac{d \left( 1 - \frac{\bar{S}}{S} \right)}{d(X/S)} = \frac{d \left[ \frac{v}{\alpha S^\theta} - \frac{1-\alpha}{\alpha} w^\theta \right]^{\frac{1}{\theta}}}{dw} \quad (\text{A.11})$$

$$= - \left[ \frac{v}{\alpha S^\theta} - \frac{1-\alpha}{\alpha} w^\theta \right]^{\frac{1}{\theta}-1} \left( \frac{v}{\alpha S^{\theta+1}} \frac{dS}{dw} + \frac{1-\alpha}{\alpha} w^{\theta-1} \right) \quad (\text{A.12})$$

Totally differentiating (A.9), and using  $dv = 0$ , yields

$$0 = \theta \left[ \alpha (S - \bar{S})^{\theta-1} + (1 - \alpha) w^\theta S^{\theta-1} \right] dS + \theta(1 - \alpha) S^\theta w^{\theta-1} dw \quad (\text{A.13})$$

$$\Leftrightarrow \frac{dS}{dw} = - \frac{(1 - \alpha) S^\theta w^{\theta-1}}{\alpha (S - \bar{S})^{\theta-1} + (1 - \alpha) w^\theta S^{\theta-1}} \quad (\text{A.14})$$

Using (A.12) and (A.14), we have:

$$-\frac{\frac{X}{S}}{\left(1 - \frac{\bar{S}}{S}\right)} \frac{d\left(1 - \frac{\bar{S}}{S}\right)}{d(X/S)} = \frac{\frac{X}{S}}{\left(1 - \frac{\bar{S}}{S}\right)} \left[ \frac{v}{\alpha S^\theta} - \frac{1 - \alpha}{\alpha} w^\theta \right]^{\frac{1}{\theta} - 1} \left( \frac{v}{\alpha S^{\theta+1}} \frac{dS}{dw} + \frac{1 - \alpha}{\alpha} w^{\theta-1} \right) \quad (\text{A.15})$$

$$= -\frac{X}{S} \left(1 - \frac{\bar{S}}{S}\right)^{-\theta} \left( \frac{v}{\alpha S^{\theta+1}} \frac{(1 - \alpha) S^\theta w^{\theta-1}}{\alpha (S - \bar{S})^{\theta-1} + (1 - \alpha) S^{\theta-1} w^\theta} - \frac{1 - \alpha}{\alpha} w^{\theta-1} \right) \quad (\text{A.16})$$

$$= -\left(\frac{X}{S}\right)^\theta \left(1 - \frac{\bar{S}}{S}\right)^{-\theta} \left[ \frac{v(1 - \alpha)}{\alpha^2 S (S - \bar{S})^{\theta-1} + \alpha(1 - \alpha) X^\theta} - \frac{1 - \alpha}{\alpha} \right] \quad (\text{A.17})$$

$$= -X^\theta (S - \bar{S})^{-\theta} \left[ \frac{v(1 - \alpha) - \alpha(1 - \alpha) S (S - \bar{S})^{\theta-1} - (1 - \alpha)^2 X^\theta}{\alpha^2 S (S - \bar{S})^{\theta-1} + \alpha(1 - \alpha) X^\theta} \right] \quad (\text{A.18})$$

$$= -X^\theta (S - \bar{S})^{-\theta} \frac{(1 - \alpha) (S - \bar{S})^\theta - (1 - \alpha) S (S - \bar{S})^{\theta-1}}{\alpha S (S - \bar{S})^{\theta-1} + (1 - \alpha) X^\theta} \quad (\text{A.19})$$

$$= (1 - \alpha) X^\theta \frac{\frac{S}{S - \bar{S}} - 1}{\alpha S (S - \bar{S})^{\theta-1} + (1 - \alpha) X^\theta}. \quad (\text{A.20})$$

Plugging this into (A.7) yields

$$\sigma(S, X) = \frac{1}{1-\theta} \left\{ 1 - \frac{\frac{X}{\bar{S}}}{\left(1 - \frac{\bar{S}}{S}\right)} \frac{d\left(1 - \frac{\bar{S}}{S}\right)}{d(X/S)} \right\}^{-1} \quad (\text{A.21})$$

$$= \frac{1}{1-\theta} \left\{ 1 + (1-\alpha)X^\theta \frac{\frac{S}{S-\bar{S}} - 1}{\alpha S (S-\bar{S})^{\theta-1} + (1-\alpha)X^\theta} \right\}^{-1} \quad (\text{A.22})$$

$$= \frac{1}{1-\theta} \left[ \frac{\alpha S (S-\bar{S})^{\theta-1} + (1-\alpha)X^\theta}{\alpha S (S-\bar{S})^{\theta-1} + (1-\alpha)X^\theta \frac{S}{S-\bar{S}}} \right] \quad (\text{A.23})$$

$$= \frac{1}{1-\theta} \left[ \frac{\alpha \left(\frac{S-\bar{S}}{X}\right)^{\theta-1} + (1-\alpha)\frac{X}{\bar{S}}}{\alpha \left(\frac{S-\bar{S}}{X}\right)^{\theta-1} + (1-\alpha)\frac{X}{S-\bar{S}}} \right] \quad (\text{A.24})$$

$$= \frac{1}{1-\theta} \left[ 1 - \frac{(1-\alpha)\frac{\bar{S}}{S}}{\alpha \left[\frac{S-\bar{S}}{X}\right]^\theta + (1-\alpha)} \right]. \quad (\text{A.25})$$

## A.2 Proof of Proposition 2

Results (8), (10), (11), (13) and (14) can easily be verified.

Proof of Result (9):

$$\frac{d\sigma}{dS} = \frac{1}{1-\theta} \frac{1-\alpha\bar{S}}{S^2} \frac{\left[ \alpha \left( \frac{\theta S}{X} \left[ \frac{S-\bar{S}}{X} \right]^{\theta-1} + \left[ \frac{S-\bar{S}}{X} \right]^\theta - 1 \right) + 1 \right]}{\left( 1 - \alpha + \alpha \left[ \frac{S-\bar{S}}{X} \right]^\theta \right)^2} > 0 \text{ for } \theta \geq 0. \quad (\text{A.26})$$

Proof of Result (12):

$$\frac{d\sigma}{d\bar{S}} = \frac{1}{\theta-1} \frac{1-\alpha}{S} \frac{\left[ \frac{\theta\alpha\bar{S}}{X} \left[ \frac{S-\bar{S}}{X} \right]^{\theta-1} + (1-\alpha) + \alpha \left[ \frac{S-\bar{S}}{X} \right]^\theta \right]}{\left( 1 - \alpha + \alpha \left[ \frac{S-\bar{S}}{X} \right]^\theta \right)^2} < 0 \text{ for } \theta \geq 0. \quad (\text{A.27})$$

### A.3 Proof of Proposition 3

The consumer requires  $m = p_S \bar{S}$  to meet her subsistence needs  $\bar{S}$ . This means that up to this level of income she is not willing to substitute  $S$  for  $X$ , i.e.  $\sigma^* = 0$ . For  $m > p_S \bar{S}$ , she faces the utility maximization problem

$$\max_{S, X} U_h(S, X) \quad \text{s.t.} \quad p_S S + p_X X \leq m . \quad (\text{A.28})$$

The Lagrangian and first-order conditions are:

$$\mathcal{L}(S, X, \mu) = \left[ \alpha (S - \bar{S})^\theta + (1 - \alpha) X^\theta \right]^{1/\theta} + \mu (m - p_S S - p_X X) \quad (\text{A.29})$$

$$\frac{\partial \mathcal{L}}{\partial S} = 0 \Leftrightarrow \alpha (S - \bar{S})^{(\theta-1)} \left[ \alpha (S - \bar{S})^\theta + (1 - \alpha) X^\theta \right]^{(1/\theta-1)} = \mu p_S \quad (\text{A.30})$$

$$\frac{\partial \mathcal{L}}{\partial X} = 0 \Leftrightarrow (1 - \alpha) X^{(\theta-1)} \left[ \alpha (S - \bar{S})^\theta + (1 - \alpha) X^\theta \right]^{(1/\theta-1)} = \mu p_X \quad (\text{A.31})$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = 0 \Leftrightarrow p_S S + p_X X = m \quad (\text{A.32})$$

From conditions (A.30) and (A.31), we obtain

$$\frac{\alpha}{1 - \alpha} \left[ \frac{(S - \bar{S})}{X} \right]^{\theta-1} = \frac{p_S}{p_X}. \quad (\text{A.33})$$

Rearranging gives

$$X = (S - \bar{S}) \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}}. \quad (\text{A.34})$$

Inserting (A.34) into (A.32) and solving for  $S$  yields the Marshallian demand function

$$S^*(m, p_S, p_X) = \frac{m + p_X \bar{S} \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}}}{p_S + p_X \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}}}. \quad (\text{A.35})$$

Inserting (A.35) into (A.34) yields the Marshallian demand function

$$X^*(m, p_S, p_X) = \frac{m - p_S \bar{S}}{p_X + p_S \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{-\frac{1}{\theta-1}}}. \quad (\text{A.36})$$

Inserting (A.35) and (A.36) into Equation (7) yields the Hicksian elasticity of substitution in the utility-optimal allocation  $(X^*, S^*)$ :

$$\sigma^* = \frac{1}{1-\theta} \left[ 1 - \frac{(1-\alpha) \bar{S} \frac{p_S + p_X \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}}}{m + p_X \bar{S} \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}}}}{(1-\alpha) + \alpha \frac{\frac{m + p_X \bar{S} \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}}}{p_S + p_X \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}}} - \bar{S}}{\frac{m - p_S \bar{S}}{p_X + p_S \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{-1}{\theta-1}}}} \right]^{\theta} \quad (\text{A.37})$$

$$= \frac{1}{1-\theta} \left[ 1 - \frac{(1-\alpha) \bar{S} \left( p_S + p_X \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}} \right)}{\left( m + p_X \bar{S} \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}} \right) \left( \alpha \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{-\theta}{\theta-1}} + (1-\alpha) \right)} \right] \quad (\text{A.38})$$

$$= \frac{1}{1-\theta} \left[ \frac{m \left( (1-\alpha) + \alpha \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{-\theta}{\theta-1}} \right)}{(1-\alpha) \bar{S} \left( p_S + p_X \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}} \right) + m \left( (1-\alpha) + \alpha \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{-\theta}{\theta-1}} \right)} \right] \quad (\text{A.39})$$

Solving the equation  $\sigma^* = 1$  for  $m = m^P$  we obtain (21).

From Equation (A.39) it can easily be seen that as income  $m$  goes to infinity, the elasticity of substitution  $\sigma^*$  approaches standard CES result in absence of a subsistence requirement:

$$\sigma^* \rightarrow \frac{1}{1-\theta} \quad \text{for } m \rightarrow \infty. \quad (\text{A.40})$$

Furthermore, the elasticity of substitution monotonically increases with income,

$$\frac{d\sigma^*}{dm} = \frac{1}{1-\theta} \left[ \frac{(1-\alpha) \bar{S} \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{2\theta+1}{\theta-1}} \left( p_X + p_S \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}} \right) \left( (1-\alpha) + \alpha \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{\theta}{\theta-1}} \right)}{\left( \alpha m + (1-\alpha) \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{\theta}{\theta-1}} \left( m + p_S \bar{S} + p_X \bar{S} \left[ \frac{\alpha}{1-\alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta-1}} \right) \right)^2} \right] > 0. \quad (\text{A.41})$$



and decreases with an increasing subsistence requirement

$$\frac{d\sigma^*}{d\bar{S}} = \frac{1}{\theta - 1} \left[ \frac{(1 - \alpha)m \left( p_S + p_X \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta - 1}} \right) \left( (1 - \alpha) + \alpha \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{\theta}{\theta - 1}} \right)}{\left( \alpha m \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{\theta}{\theta - 1}} + (1 - \alpha) \left( m + p_S \bar{S} + p_X \bar{S} \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta - 1}} \right) \right)^2} \right] < 0. \quad (\text{A.42})$$

#### A.4 Proof of Proposition 4

The cross-price derivatives of the Marshallian demand functions for  $S$  and  $X$  are obtained from Equations (A.35) and (A.36) and have the following properties:

$$\frac{dS^*}{dp_X} = \frac{\theta}{\theta - 1} \left[ \frac{(p_S \bar{S} - m) \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta - 1}}}{\left( p_S + p_X \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{1}{\theta - 1}} \right)^2} \right] \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad \text{for } \theta \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (\text{A.43})$$

$$\frac{dX^*}{dp_S} = \frac{1}{\theta - 1} \left[ \frac{\bar{S} \left( (1 - \theta)p_X + p_S \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{-1}{\theta - 1}} \right) - \theta m \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{-1}{\theta - 1}}}{\left( p_X + p_S \left[ \frac{\alpha}{1 - \alpha} \frac{p_X}{p_S} \right]^{\frac{-1}{\theta - 1}} \right)^2} \right], \quad (\text{A.44})$$

with

$$\frac{dX^*}{dp_S} \left\{ \begin{matrix} \rightarrow \infty \\ = -(1 - \alpha) \frac{\bar{S}}{p_X} < 0 \\ < 0 \end{matrix} \right\} \quad \text{for } \theta \left\{ \begin{matrix} \rightarrow 1 \\ = 0 \\ < 0 \end{matrix} \right\}. \quad (\text{A.45})$$

Solving the equation  $dX^*/dp_S = 0$  for  $m$ , we obtain (25). It is further easy to verify that  $\frac{d^2 X^*}{dp_S dm} > 0$  for  $\theta > 0$ .

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Leuphana Universität Lüneburg

Institut für Volkswirtschaftslehre

Postfach 2440

D-21314 Lüneburg

Tel.: ++49 4131 677 2321

email: brodt@leuphana.de

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