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Likelihood-based panel cointegration test in the presence of a linear time trend and cross-sectional dependence

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Abstract

This paper proposes a new likelihood-based panel cointegration rank test which extends the test of Örsal and Droge (2012) (henceforth Panel SL test) to allow for cross-sectional dependence. The dependence is modelled by unobserved common factors which affect the variables in each cross-section through heterogeneous loadings. The common components are estimated following the panel analysis of nonstationarity in idiosyncratic and common components (PANIC) approach of Bai and Ng (2004) and the estimates are subtracted from the observations. The cointegrating rank of the defactored data is then tested by the Panel SL test. A Monte Carlo study demonstrates that the proposed testing procedure has reasonable size and power properties in finite samples.

JEL classification: C12, C15, C33
Keywords: panel cointegration rank test, cross-sectional dependence, common factors, likelihood-ratio, time trend

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1 Introduction

The cointegration methodology has become a principal tool in investigating the long-run relationships between non-stationary economic variables over the past 25 years. Since many macroeconomic variables exhibit trending behaviour, unit root and cointegration tests that accommodate a polynomial time trend in the data generating process have been developed. The proposed tests are not without limitations. For example, in the Johansen's likelihood-ratio (LR) test for the cointegrating rank of a system of variables (Johansen, 1995) the distribution of the test statistic under the null hypothesis depends on whether a deterministic trend term actually exists in the data generating process or not. To overcome the difficulty of deciding upon the correct asymptotic distribution in such cases, Saikkonen and Lütkepohl (2000) proposed subtracting GLS estimates of the deterministic terms from the observed data and applying the cointegrating rank test to the trend-adjusted data. In a Monte Carlo study they showed that their test outperforms the Johansen's LR-type test allowing for a linear time trend. However, both types of tests have low power when a near-unit root component is present in the process.

In practice, the power of unit root and cointegration tests might be limited for a single cross-section and can thus be improved upon by considering panel data. The approach of Saikkonen and Lütkepohl (2000) has been extended to the panel framework by Örsal and Droge (2012). Following Larsson et al. (2001), the test statistic of the panel Saikkonen-Lütkepohl (SL) test is computed by standardising the average of the individual LR trace statistics over the cross-sections. Under the null hypothesis the test statistic converges to a standard normal random variable provided that the number of time observations T and the number of cross-sections N tend to infinity sequentially. A critical assumption for the standardisation of the average of the LR trace statistics is the independence between cross-sections.

However, cross-sectional independence, although theoretically convenient for the asymptotic analysis, may be an unrealistic and highly restrictive assumption in practice. Panel unit root and cointegration tests relying on this assumption are known to suffer from severe size distortions when applied to panels in which cross-sectional dependence is present (see Gengenbach et al. (2006), Wagner and Hlouskova (2010), Carrion-i Silvestre and Surdeanu (2011)).

Several methods have been proposed to model the cross-sectional dependence. Groen and Kleibergen (2003) developed a test for the cointegrating rank considering panel vector error correction models (VECMs), where they introduced correlation between the cross-sections through the disturbance covariance matrix. This test involves iterative generalised method of moments estimation. Miller (2010) proposed a panel likelihood-based cointegration rank test in which he followed the non-linear instrumental variables approach of Chang (2002) to cope with the cross-sectional dependence. Both tests are relatively complex to compute and require a large time dimension of the panel while keeping the cross-sectional dimension fixed.

A large strand in the recent unit root and cointegration literature models the cross-sectional dependence in large T, large N panels by unobserved common factors. The PANIC approach of Bai and Ng (2004) extracts the common factors and their loadings by principal components and essentially decomposes the observed data into estimates of the unobserved common and idiosyncratic components. Gengenbach et al. (2006) and Bai and Carrion-i Silvestre (2013), among others, adopted the PANIC methodology and proposed residual-based cointegration tests for the case of dependent panels; Carrion-i Silvestre and Surdeanu (2011) employed it to develop a panel cointegration rank test. In a recent unpublished study, Callot (2010) introduced unobserved common factors into the likelihood-based panel framework. He, on the other hand, followed the approach of Pesaran (2006) and Dees et al. (2007) to account for the influence of the factors by cross-sectional averages of the observed variables and proposed two panel rank tests based on the p-values of the bootstrapped individual LR trace statistics. To the best of our knowledge, however, the common factor framework has not yet been utilised to extend likelihood-based panel cointegration tests to the case of cross-sectional dependence in the sense of applying a rank test to defactored data.

To close this gap, we extend the panel SL test of Örsal and Droge (2012) to allow for cross-sectional dependence by including the unobserved common factors in the equation for the observed data. In our setting the common factors may be integrated of order zero or one, or a combination of both, and potentially cointegrated.

We adopt the PANIC approach of Bai and Ng (2004) to extract the common factors and their loadings by principal components from the first differenced and demeaned data. The estimated common components are then subtracted from the observed data, thus removing the cross-sectional dependence from the panel. Testing for no cointegration of the idiosyncratic components is performed by directly applying the Panel SL test to the defactored data. Should the null hypothesis be rejected, testing for a cointegrating rank greater than or equal to one proceeds by a modification of the sequential procedure of Johansen (1995). First the common idiosyncratic stochastic trends under the new null hypothesis are estimated from the defactored data, after which they are tested for no cointegration by the Panel SL test. A Monte Carlo study demonstrates that the proposed procedure maintains reasonable size and high power in almost all experimental settings considered, provided that the common trends are selected by the estimator of the right null space of the cointegrating relations computed as in Johansen (1995) from the defactored data.

The remainder of the paper is organised as follows. Section 2 introduces the model and the relevant assumptions regarding the idiosyncratic and the common components. Section 3 describes the estimation of the common components, establishes the properties of the Panel SL test for no cointegration on defactored data and outlines the procedure for testing for the cointegrating rank. The finite sample properties of the test analysed by means of Monte Carlo simulations are presented in Section 4 and Section 5 concludes. All proofs are deferred to the Appendix.

The following notation is used throughout the paper. The superscript ^{cd} denotes the observed cross-sectionally dependent processes, the star symbol * signifies their defactored counterparts and quantities computed from them, while the tilde $\tilde{}$ is reserved for GLS-detrended processes. L and Δ denote the lag and differencing operators respectively. $I(d)$ denotes a process which is integrated of order d, and $W(s)$ stands for a standard multivariate Wiener process of a suitable dimension. Convergence in distribution is signified through $\Rightarrow$, while $\xrightarrow{p}$ denotes convergence in probability and $\sim$ stands for asymptotic equivalence. For an $(n \times n)$ matrix A, $\mathrm{tr}(A)$, $\|A\| = [\mathrm{tr}(A'A)]^{1/2}$ and $\lambda_i(A)$, $i = 1, \ldots, n$, denote its trace, Euclidean norm and eigenvalues respectively, while $\mathrm{rk}(B)$ stands for the rank of an arbitrary $(m \times n)$, $m > n$, matrix B. If B is of full column rank n, we denote its orthogonal complement by $B_\perp$; that is, $B_\perp$ is an $(m \times (m-n))$ matrix of full column rank such that $(B, B_\perp)$ is of full rank m and $B_\perp' B = 0$. The orthogonal complement of a non-singular square matrix is zero and the orthogonal complement of zero is an identity matrix of suitable dimension; an $(n \times n)$ identity matrix is denoted by I_n. We finally let $C_{NT} = \min\left(\sqrt{N}, \sqrt{T}\right)$ and $M < \infty$ be a generic constant which is independent of the dimensions of the panel N and T.

2 Model setting

We consider a panel data set consisting of N cross-sections (individuals) observed over T time periods. For each individual i $(i = 1, \ldots, N)$ the observed m-dimensional time series $Y_{it}^{cd} = (Y_{1,it}^{cd}, \ldots, Y_{m,it}^{cd})'$, $t = 1, \ldots, T$, is generated by a VAR(p_i) process Y_{it} with a linear time trend plus common components $\Lambda_i' F_t$ which drive the cross-sectional dependence:

$$
\begin{aligned}
Y_{it}^{cd} &= Y_{it} + \Lambda_i' F_t, \quad i = 1, \ldots, N, \ t = 1, \ldots, T, \\
Y_{it} &= \mu_{0i} + \mu_{1i} t + X_{it}, \\
X_{it} &= A_{i1} X_{i,t-1} + \ldots + A_{i,p_i} X_{i,t-p_i} + \varepsilon_{it}, \\
(1-L) F_t &= C(L) u_t.
\end{aligned}
\tag{1}
$$

Here μ_{0i} and μ_{1i} are unknown m-dimensional parameter vectors, p_i is the lag order of the VAR process for the ith cross-section and $A_{i1}, \ldots, A_{i,p_i}$ are unknown $(m \times m)$ coefficient matrices. The observed series for each cross-section is assumed to be influenced at any time instance t by a $(k \times 1)$ vector of unobserved common factors F_t through individual-specific factor loading matrices Λ_i of dimension $(k \times m)$. In the specification of the process for the common factors F_t above, $C(L) = \sum_{j=0}^{\infty} C_j L^j$, where the rank of $C(1)$ is k_1, $0 \leq k_1 \leq k$. This allows for k_1 common stochastic trends and $k_0 = k - k_1$ stationary factors. The idiosyncratic errors ε_{it} are assumed to be serially and cross-sectionally independent and thus the dependence between the individuals is driven solely by the unobserved common factors F_t. The components of the process X_{it} are assumed to be integrated of order at most one and cointegrated with cointegrating rank r_i, $0 \leq r_i \leq m$. This implies the following VECM for X_{it}:

$$
\Delta X_{it} = \Pi_i X_{i,t-1} + \sum_{j=1}^{p_i - 1} \Gamma_{ij} \Delta X_{i,t-j} + \varepsilon_{it}, \quad t = p_i + 1, \ldots, T,
$$

where $\Gamma_{ij} = -\left(A_{i,j+1} + \ldots + A_{p_i}\right)$. The $(m \times m)$ matrix $\Pi_i = -\left(I_m - A_{i1} - \ldots - A_{i,p_i}\right)$ has rank $r_i \leq m$ and can therefore be represented as $\Pi_i = \alpha_i \beta_i'$ with α_i and β_i being full rank $(m \times r_i)$ matrices. For further use let $A_i(L) \equiv I_m - \sum_{i=1}^{p_i - 1} A_i L^i$ and $\Gamma_i = I_m - \sum_{j=1}^{p_i - 1} \Gamma_{ij}$.

We make the following assumptions.

Assumption 1 Integrating properties of the idiosyncratic components:

(a) $|A_i(z)| = 0$ implies that either $|z| > 1$ or $z = 1$ for each $i = 1, \ldots, N$.

(b) The matrix $\alpha'_{i,\perp}\Gamma_i\beta_{i,\perp}$ has full rank $(m-r_i)$ and $\left\|(\alpha'_{i,\perp}\Gamma_i\beta_{i,\perp})^{-1}\right\| \leq M$ for each $i = 1, \dots, N$.

Assumption 2 Common factors:

(a) $u_t \sim iid(0, \Sigma_u)$, $\mathbb{E}||u_t||^4 \leq M$.

(b) $Var(\Delta F_t) = \sum_{j=0}^{\infty} C_j \Sigma_u C'_j > 0$.

(c) $\sum_{j=0}^{\infty} j||C_j|| < M$.

(d) $C(1)$ has rank k_1, $0 \leq k_1 \leq k$.

Assumption 3 Factor loadings:

(a) Λ_i is deterministic and $||\Lambda_i|| \leq M$, or Λ_i is stochastic and $\mathbb{E}||\Lambda_i||^4 \leq M$ for each $i = 1, \dots, N$.

(b) $N^{-1}\sum_{i=1}^{N} \Lambda_i\Lambda'_i \xrightarrow{p} \Sigma_{\Lambda_i}$ as $N \to \infty$, where Σ_{Λ_i} is a non-random positive definite $(k \times k)$ matrix for each $i = 1, \dots, N$.

Assumption 1 (a) gives a necessary and sufficient condition for the processes $\beta'_i X_{it}$ and $\beta'_{i,\perp}X_{it}$ to be integrated of order zero and one respectively, and part (b) is necessary for the proof of the joint limiting distribution in Theorem 3.3. The latter two assumptions are standard in the factor models literature. The invertible limit in probability of $N^{-1}\sum_{i=1}^{N} \Lambda_i\Lambda'_i$ implies that each factor contributes to the variance of at least one of the variables in Y_{it}^{cd}, resulting in strong cross-sectional dependence.

Assumption 4 Idiosyncratic errors:

The idiosyncratic errors are assumed to be serially and cross-sectionally independent and normally distributed, i.e. $\varepsilon_{it} \sim N_m(0, \Omega_i)$, where Ω_i is some non-random positive definite matrix and $i = 1, \dots, N$.

Assumption 5 Independence of common factors, factor loadings and idiosyncratic errors:

Λ_i, u_t and ε_{it} are mutually independently distributed across i and t.

Assumption 6 Number of common factors:

The number of the common factors k is assumed to be known. Alternatively, consistent estimates of the number of the common factors may be obtained as proposed by Bai and Ng (2002) or Onatski (2010).

3 The Panel SL cointegration rank test

Our aim is to extend the likelihood-based panel cointegration rank test of Örsal and Droge (2012) to the case of cross-sectional dependence. Since the Panel SL test statistic is calculated as the standardised average of the individual LR test statistics, the test relies on the assumption of independence between cross-sections. In order to apply the Panel SL test, the influence of the common factors needs to be removed as a first step. Following the PANIC approach of Bai and Ng (2004), the common factors are extracted by the method of principal components from the first differenced and demeaned observed data. The cumulated estimates of the common components are then subtracted from the observed data yielding cross-sectionally independent observations.

3.1 Defactoring the data

We apply the defactoring procedure for the linear trend case of the PANIC approach proposed by Bai and Ng (2004). Introducing the following notation for the time averages:

$$\overline{\Delta Y_i^{cd}} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta Y_{it}^{cd}, \quad \overline{\Delta X_i} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta X_{it} \text{ and } \overline{\Delta F} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta F_t,$$

we take equation (1) in first differences and demean it in order to remove the linear trend term, which results in obtaining

$$\Delta Y_{it}^{cd} - \overline{\Delta Y_i^{cd}} = \Lambda_i' \left(\Delta F_t - \overline{\Delta F}\right) + \left(\Delta X_{it} - \overline{\Delta X_i}\right). \tag{2}$$

Letting

$$y_{it} = \Delta Y_{it}^{cd} - \overline{\Delta Y_i^{cd}}, \quad f_t = \Delta F_t - \overline{\Delta F} \text{ and } x_{it} = \Delta X_{it} - \overline{\Delta X_i},$$

(2) can be written as

$$y_{it} = \Lambda_i' f_t + x_{it}. \tag{3}$$

Stacking the observations for each cross-section over time, we obtain

$$y_i = f\Lambda_i + x_i, \quad i = 1, \ldots, N, \tag{4}$$

where

$$y_i = (y_{i2}, \ldots, y_{iT})', \; f = (f_2, \ldots, f_T)' \text{ and } x_i = (x_{i2}, \ldots, x_{iT})'.$$

The combined model for all cross-sections then reads

$$y = f\Lambda + x, \tag{5}$$

where

$$y = (y_1, \ldots, y_N), \; \Lambda = (\Lambda_1, \ldots, \Lambda_N) \text{ and } x = (x_1, \ldots, x_N).$$

The first-differenced and demeaned common factors f and the factor loadings Λ can now be extracted by the method of principal components (PC) applied to the $((T-1)\times Nm)$-dimensional data matrix y. The PC estimator $\hat{f}$ is obtained as $\sqrt{T-1}$ times the normalised eigenvectors corresponding to the k largest eigenvalues of the moment matrix yy'. The factor loading estimates are computed as $\hat{\Lambda} = \hat{f}'y/(T-1)$. The estimates of the common factors are recovered by cumulating $\hat{f}_t$:

$$\hat{F}_t = \sum_{s=2}^{t} \hat{f}_s \text{ for } t = 2, \ldots, T, \text{ with } \hat{F}_1 = 0. \tag{6}$$

The observed data can now be defactored by subtracting the common component estimates, thereby removing the cross-sectional dependence:

$$Y_{it}^* = Y_{it}^{cd} - \hat{\Lambda}_i' \hat{F}_t, \tag{7}$$

or by recovering the idiosyncratic components as

$$\hat{X}_{it} = \sum_{s=2}^{t} \left(y_{is} - \hat{\Lambda}_i' \hat{f}_s \right) \text{ for } t = 2, \ldots, T, \text{ with } \hat{X}_{i1} = 0. \tag{8}$$

Bai and Ng (2004) have shown that the estimates of the common factors consistently estimate the space spanned by the true factors provided $T, N \to \infty$ simultaneously. We note that using the estimates of the idiosyncratic components $\hat{X}_{it}$ instead of Y_{it}^* for testing for no cointegration in the procedure described below yields equivalent results, since they only differ in the deterministic trend component. For details we refer to the Appendix. Both types of defactored data can be used for testing the cointegrating rank too, however employing Y_{it}^* requires an additional step of OLS detrending in some cases, as outlined in Section 3.3, while employing $\hat{X}_{it}$ does not.

3.2 The Panel SL test for no cointegration

This section describes the procedure for testing the null hypothesis of no cointegration in the unobserved process X_{it} based on the defactored data Y_{it}^* (or equivalently $\hat{X}_{it}$). Having removed the cross-sectional dependence in the way described above, we apply the methodology of Örsal and Droge (2012). The first step consists of GLS detrending of the defactored series as suggested by Saikkonen and Lütkepohl (2000). We refer to the latter work for more details on the estimation procedure.

It should be noted, however, that the GLS detrending yields estimates of the deterministic terms with the necessary consistency rates only under the null hypothesis $H_0 : r_i = r = 0, \forall i = 1, \ldots, N$ which implies that $\alpha_i = \beta_i = 0$. The reason is that, although the estimation of the space spanned by the common components is consistent as T and N grow large, defactoring the observed data introduces both a deterministic and a stochastic trend to every variable in the system. The stochastic trend diverges at rate $O_p(\sqrt{T/N})$ rather than the usual $O_p(\sqrt{T})$; more details are given in the Appendix. This stochastic trend is the reason why the cointegrating vectors β_i cannot be estimated with the usual consistency rate $O_p(T^{-1})$ from Y_{it}^* (or $\hat{X}_{it}$) by the method of Johansen (Johansen, 1995, pp. 89-92), unless the relative expansion rate between N and T is $T/N \to 0$ as $T, N \to \infty$ simultaneously, which suppresses the influence of the unwanted stochastic trend in the limit. In order to establish the joint limiting distribution of the Panel SL test, however, $N/T \to 0$ is required as $T, N \to \infty$ simultaneously. For this reason we first concentrate on the extension of the Panel SL test to defactored data only for testing the null hypothesis of no cointegration, as in this case estimation of the cointegrating relations is not required. In order to test for the cointegrating rank, a slightly more involved sequential testing procedure has to be followed. This is outlined in Section 3.3.

Denoting the defactored and detrended observations by $\tilde{X}^*_{it}$, the panel cointegration test is applied to the following VECM:

$$\Delta\tilde{X}^*_{it} = \Pi_i\tilde{X}^*_{i,t-1} + \sum_{j=1}^{p_i-1}\Gamma_{ij}\Delta\tilde{X}^*_{i,t-j} + \epsilon_{it}, \quad \text{for } i = 1,\ldots,N;\ t = p_i+1,\ldots,T, \tag{9}$$

where $\tilde{X}^*_{it} = \tilde{Y}^*_{it} - \tilde{\mu}^+_{i0} - \tilde{\mu}^+_{i1}t$ and $\tilde{\mu}^+_{i0}$, $\tilde{\mu}^+_{i1}$ denote the GLS estimates of the intercept and trend parameters of the defactored data respectively[1].

Our aim is to test the null hypothesis of no cointegration, i.e. $r_i = \text{rk}(\Pi_i) = 0$ for each $i = 1,\ldots,N$:

$$H_0 : r_i = r = 0, \forall i \ \text{ versus } \ H_1 : r_i > 0 \text{ for some i.} \tag{10}$$

For each cross-section we compute the GLS-based LR trace statistic from the defactored data as

$$\text{LR}^{\text{SL}*}_{\text{trace}_{\text{iT}}}(\text{r}) = -T\sum_{j=r+1}^{m}\ln\left(1-\hat{\lambda}^*_{ij}\right) \tag{11}$$

with $r = 0$. Here $\hat{\lambda}^*_{i1} > \ldots > \hat{\lambda}^*_{im}$ denote the ordered solutions of the eigenvalue problem defined in Johansen (1995, pp. 90-93) for the VECM (9) (see (A.15) in the Appendix). It is worth mentioning that the individual GLS-based LR statistics in (11) can be computed with the free software JMulTi.

The asymptotic distribution of the individual $\text{LR}^{\text{SL}}_{\text{trace}_{\text{iT}}}(\text{r})$ statistics calculated as above, but from cross-sectionally independent data, has been derived by Saikkonen and Lütkepohl (2000). More specifically, they have derived the distribution of the LR trace statistic based on GLS detrended data for a single cross-section $\tilde{X}_t$, which holds for each unit in a cross-sectionally independent panel under the null hypothesis $\text{rk}(\Pi_i) = r$. It is given by

$$\text{LR}^{\text{SL}}_{\text{trace}_{\text{iT}}}(\text{r}) \Rightarrow\ Z_d \text{ as } T \to \infty,$$

with

$$Z_d = \text{tr}\left[\left(\int_0^1 W_*(s)dW_*(s)'\right)'\left(\int_0^1 W_*(s)W_*(s)'ds\right)^{-1}\left(\int_0^1 W_*(s)dW_*(s)'\right)\right], \tag{12}$$

where $W_*(s) = W(s) - sW(1)$ is a $d-$dimensional Brownian bridge with $d = m-r$, $dW_*(s) = dW(s) - dsW(1)$ and $W(s)$ is a standard $d-$dimensional Brownian motion. When testing the null of no cointegration, $d = m$.

[1]The linear time trend term $\mu^+_{i0} + \mu^+_{i1}t$ of the defactored data comprises the trend term of the observed process $\mu_{i0} + \mu_{i1}t$ plus an additional linear trend term arising from the defactoring procedure. Please refer to the Appendix for details.

Denoting $p = \max\{p_i | 1 \leq i \leq N\}$, we extend this result to the case of defactored data as follows (proofs are outlined in the Appendix).

Theorem 3.1. *Under the null hypothesis of no cointegration and assuming that m and p remain fixed as $T, N \to \infty$ simultaneously, for each cross-section $i = 1, \ldots, N$ it holds that:*

$$\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace_{iT}}}(0) = \mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0) + O_p\left(C_{NT}^{-1}\right), \tag{13}$$

where $C_{NT} = \min\left(\sqrt{N}, \sqrt{T}\right)$. Under the alternative, $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace_{iT}}}(0)$ diverges to $+\infty$.

As an immediate result of the theorem we obtain the limiting distribution of $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace_{iT}}}(0)$:

Corollary 3.1. *Under the assumptions of the theorem the limiting distribution of $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace_{iT}}}(0)$ as $T, N \to \infty$ simultaneously is the same as that of $\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0)$ as $T \to \infty$ and is given by (12) with $d = m$.*

In this way we have established the asymptotic equivalence of the individual LR trace statistics $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace_{iT}}}(0)$ and $\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0)$ under $H_0 : r_i = r = 0$ for each $i = 1, \ldots, N$.

The Panel SL test statistic of Örsal and Droge (2012) is obtained by standardising the cross-sectional average of the individual LR trace statistics by the moments of the limiting random variable Z_d. Let

$$\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace_{NT}}}(0) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0),$$

and

$$\overline{\mathrm{LR}}^{\mathrm{SL}*}_{\mathrm{trace_{NT}}}(0) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace_{iT}}}(0).$$

In the next theorem we establish that, under the null hypothesis and when $\sqrt{N}/T \to 0$ as T and N grow jointly to infinity, the cross-sectional averages $\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace_{NT}}}(0)$ and $\overline{\mathrm{LR}}^{\mathrm{SL}*}_{\mathrm{trace_{NT}}}(0)$ are asymptotically equivalent when normalised by $\sqrt{N}$.

Theorem 3.2. *Under the null hypothesis of no cointegration and assuming that m and p remain fixed as $T, N \to \infty$ simultaneously, it holds that*

$$\sqrt{N}\ \overline{\mathrm{LR}}^{\mathrm{SL}*}_{\mathrm{trace_{NT}}}(0) = \sqrt{N}\ \overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace_{NT}}}(0) + O_p\left(\frac{\sqrt{N}}{T}\right) + O_p\left(C_{NT}^{-1}\right).$$

Corollary 3.2. *Under the assumptions of the theorem and assuming that $\frac{\sqrt{N}}{T} \to 0$ as $T, N \to \infty$ simultaneously,*

$$\sqrt{N}\ \overline{\mathrm{LR}}^{\mathrm{SL}*}_{\mathrm{trace_{NT}}}(0) = \sqrt{N}\ \overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace_{NT}}}(0) + o_p(1).$$

The last step of the calculation of the Panel SL test statistic is to standardise $\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}_{\mathrm{NT}}}(0)$ by the mean and the standard deviation of the asymptotic trace statistic Z_d resulting in:

$$\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}}} = \frac{\sqrt{N}\left(\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}_{\mathrm{NT}}}(0) - \mathbb{E}\left(Z_d\right)\right)}{\sqrt{\mathrm{Var}\left(Z_d\right)}}, \tag{14}$$

where $\mathbb{E}\left(Z_d\right)$ and $\mathrm{Var}\left(Z_d\right)$ are the mean and variance of the individual asymptotic trace statistic Z_d, with $d = m$ when testing $H_0 : r_i = r = 0$, $\forall i$. Theorem 2 of Örsal and Droge (2012) states that under the null hypothesis the asymptotic distribution of the panel cointegration statistic $\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}}}$ is standard normal as $T \to \infty$, followed by $N \to \infty$. In order to employ the defactored data for testing, however, we need to establish the limiting distribution of $\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}}}$ as $T, N \to \infty$ simultaneously. This is the statement of the next theorem.

Theorem 3.3. *Under the null hypothesis of no cointegration and assuming that m and p remain fixed as $T, N \to \infty$ simultaneously with $\frac{N}{T} \to 0$, it holds that*

$$\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}}} = \frac{\sqrt{N}\left(\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}_{\mathrm{NT}}}(0) - \mathbb{E}\left(Z_d\right)\right)}{\sqrt{Var\left(Z_d\right)}} \Rightarrow N(0,1). \tag{15}$$

Corollary 3.3. *Under the assumptions of the theorem, for the Panel SL test statistic based on the defactored data it holds that*

$$\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}*}_{\mathrm{trace}}} = \frac{\sqrt{N}\left(\overline{\mathrm{LR}}^{\mathrm{SL}*}_{\mathrm{trace}_{\mathrm{NT}}}(0) - \mathbb{E}\left(Z_d\right)\right)}{\sqrt{Var\left(Z_d\right)}} = \Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}}} + o_p(1),$$

and hence

$$\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}*}_{\mathrm{trace}}} \Rightarrow N(0,1).$$

Corollary 3.3 presents the main result of the paper: inference regarding the absence of cointegration among the components of the unobserved X_{it} can be made by the Panel SL test statistic computed from the defactored data Y^*_{it} (or equivalently $\hat{X}_{it}$), provided that $N/T \to 0$ as $T, N \to \infty$ simultaneously.

The test is one-sided and rejects H_0 at significance level α if

$$\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}*}_{\mathrm{trace}}}(r) > z_{1-\alpha},$$

with $z_{1-\alpha}$ being the $(1-\alpha)$ quantile of the standard normal distribution.

Approximations of the first two moments of Z_d based on large-T simulations are available for $d = 1, ..., 12$ in Örsal and Droge (2012). Alternatively, these moments may be obtained by response surface techniques; for computational details we refer to Trenkler (2008). Unpublished moments of the Z_d statistic have been provided by Carsten Trenkler and are presented in Table 1.

The simulation study of Örsal and Droge (2012) reveals that standardising the $\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}_{\mathrm{NT}}}(\mathrm{r})$ statistic by the response surface moments results in better size properties of the Panel SL test compared to standardising by the moments based on large-T simulations. We thus employ the response surface moments from Table 1 to analyse the performance of the panel SL test on defactored data.

Table 1: Simulated first two moments of Z_d via response surface approach.

$d = m - r$	$\mathbb{E}(Z_d)$	$\text{Var}(Z_d)$	$d = m - r$	$\mathbb{E}(Z_d)$	$\text{Var}(Z_d)$
1	2.689	4.396	7	99.036	147.468
2	8.924	13.725	8	129.025	193.158
3	19.011	28.501	9	163.003	241.215
4	33.036	48.837	10	200.971	297.598
5	51.023	75.430	11	242.960	360.760
6	73.042	107.953	12	289.002	428.035

3.3 Testing for the cointegrating rank

Since the observed series Y_{it}^{cd} is represented as the sum of two unobserved stochastic components, testing for its cointegrating rank is not a trivial task. As the common factors are themselves allowed to be integrated and possibly cointegrated, common stochastic trends in Y_{it}^{cd} may result from: *(i)* idiosyncratic stochastic trends shared by the variables within X_{it}, *(ii)* common stochastic trends shared by the factors in F_t if $k > 1$, resulting in cross-unit cointegration, and *(iii)* both of these sources. Decomposing the observed series into idiosyncratic and common components by the PANIC methodology allows us to investigate each of these points separately.

The cointegrating rank of X_{it}, which is the primary focus of this paper, can be determined from the defactored data by the Panel SL test following a modified version of the sequential testing procedure of Johansen (1988). The idea underlying the testing of $H_0 : r_i \leq r$, $\forall i$, for $1 \leq r \leq m-1$, is to select the "best candidates" for the $d = m - r$ common stochastic trends in each system and to test them for no cointegration by the Panel SL test. The procedure can be briefly summarised as follows:

1. Test the defactored data for no cointegration by the Panel SL test.
2. If the null hypothesis $H_0 : r_i = 0$, $\forall i$ is rejected, assume cointegrating rank $r_i = 1$ for at least one cross-section, i.e. $H_0 : \bar{r} = 1$ where $\bar{r} = \max\{r_i | 1 \leq i \leq N\}$. This translates into having at most $d = m - \bar{r}$ different stochastic trends.
3. Compute a consistent estimate of the $(m \times d)-$dimensional space orthogonal to the cointegrating relations $\beta_{i,\perp}$ from the defactored data. Select the hypothesized stochastic trends as $\hat{\beta}_{i,\perp}' Y_{it}^*$ (or $\hat{\beta}_{i,\perp}' \hat{X}_{it}$).
4. Return to Step 1 and test the panel of d-variate processes $\hat{\beta}_{i,\perp}' Y_{it}^*$ (or $\hat{\beta}_{i,\perp}' \hat{X}_{it}$) for no cointegration. Failing to reject the null hypothesis leads to the conclusion that the maximum cointegrating rank over the cross-sections is $\bar{r}$. If the null hypothesis is rejected again, increase the hypothesized rank $\bar{r}$ by one and repeat steps 2-4 until the null is not rejected or until $H_0 : \bar{r} = m - 1$ is tested.

We consider two different estimators of $\beta_{i,\perp}$. The first one is the principal components estimator proposed by Stock and Watson (1988). It is computed as the eigenvectors corresponding to the d largest eigenvalues of $\frac{1}{T^2}\sum_{t=2}^{T}\hat{X}_{it}\hat{X}_{it}'$, (or, alternatively, of $\frac{1}{T^2}\sum_{t=2}^{T}Y_{it}^{*\tau}\left(Y_{it}^{*\tau}\right)'$, where $Y_{it}^{*\tau}$ is the projection of Y_{it}^{*} on the space spanned by $(1,t)$). This estimator is employed by Carrion-i Silvestre and Surdeanu (2011) in their PMSB cointegration rank test. The second estimator is obtained as the right null space of the estimator of β_i computed by a slightly modified version of the method of Johansen. Suppressing the index i for brevity, recall that in this case $\hat{\beta}$ is estimated as the eigenvectors corresponding to the r smallest eigenvalues of the eigenvalue problem

$$\left|\lambda S_{11} - S_{10}S_{00}^{-1}S_{01}\right| = 0.$$

Since scaling a matrix by a scalar does not affect its eigenvectors, we compute our estimator from the eigenvalue problem

$$\left|\lambda\frac{1}{T}\hat{S}_{11} - \hat{S}_{10}\hat{S}_{00}^{-1}\hat{S}_{01}\right| = 0,$$

where the moment matrices $\hat{S}_{jk}$, $j,k \in \{0,1\}$, are calculated as in Johansen (1995, pp. 96-97) from the defactored data allowing for a time trend. Convergence in probability of these moment matrices to their counterparts based on the cross-sectionally independent data can be established as in Lemma A.4 in the Appendix.

It is worth noting that although both estimators are no longer superconsistent because they are computed from the defactored data, this will not alter the convergence properties of the individual LR trace test statistics. Instead of the usual $O_p(T^{-1})$ consistency rate, we get

$$\hat{\beta}_{i,\perp} - \beta_{i,\perp}C_i = O_p(C_{NT}^{-1}),\ \forall i,$$

for some full-rank matrices C_i (see also Carrion-i Silvestre and Surdeanu, 2011, pp. 33). This estimation error affects the selection of the stochastic trends as

$$\hat{\beta}_{i,\perp}'Y_{it}^{*} = C_i'\beta_{i,\perp}'Y_{it}^{*} + \left(\hat{\beta}_{i,\perp} - \beta_{i,\perp}C_i\right)'Y_{it}^{*}.$$

From the above representation it is easy to see that the GLS-detrended counterpart of the error term $\left(\hat{\beta}_{i,\perp} - \beta_{i,\perp}C_i\right)'Y_{it}^{*}$ will enter the cross-product matrices S_{jk}^{*} for the derivation of the individual $\mathrm{LR}_{\mathrm{trace_{iT}}}^{\mathrm{SL}*}(0)$ statistics as an error term of order at most $O_p(C_{NT}^{-1})$ (see Lemma A.4 in the Appendix).

It should also be noted that the above testing procedure is most powerful when all cross-sectional units have the same cointegrating rank.

The unobserved common factors can be tested for unit roots by the $ADF_{\hat{F}}^{\tau}$ or by the MQ_f^{τ} or MQ_c^{τ} tests proposed by Bai and Ng (2004). If these turn out to be stationary, then we conclude that the cointegrating properties of X_{it} determine those of the observed series Y_{it}^{cd}. Should the common factors be classified as non-stationary, then their cointegrating rank in the case $k > 1$ can be tested by the GLS-based LR trace test of Saikkonen and Lütkepohl (2000) applied to the estimates $\hat{F}_t$. The cointegrating rank of the common component for the i-th cross-sectional unit then depends also on the individual loading Λ_i. If the idiosyncratic components turn out to be $I(0)$, then the cointegrating rank of Y_{it}^{cd} is determined by that of the common factors.

As noted by Gengenbach et al. (2006), however, it is very unlikely that in the case of $I(1)$ factors and $I(1)$ idiosyncratic components there exists a cointegrating vector β for Y_{it}^{cd} which simultaneously eliminates both sources of nonstationarity.

4 Monte Carlo simulation study

4.1 Data generating process

The data generating process (DGP) considered for the simulation study is an extension of the three-variate VAR(1) Toda process (Toda, 1994, 1995) to which common factors have been added. This process has been chosen to maintain consistency and to enable comparison of the results with those of Saikkonen and Lütkepohl (2000) and Örsal and Droge (2012).

The general form of the DGP is:

$$Y_{it} = \mu_{0i} + \mu_{1i}t + X_{it} + \Lambda_i' F_t, \tag{16}$$

$$X_{it} = \begin{pmatrix} \psi_a & 0 & 0 \\ 0 & \psi_b & 0 \\ 0 & 0 & 1 \end{pmatrix} X_{i,t-1} + \varepsilon_{it}, \tag{17}$$

$$\varepsilon_{it} \sim iidN \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \theta_1 & \theta_2 \\ \theta_1 & 1 & \theta_3 \\ \theta_2 & \theta_3 & 1 \end{pmatrix} \right], \tag{18}$$

$$F_t = BF_{t-1} + u_t, \quad u_t \sim N\left(0, \sigma_F^2\right). \tag{19}$$

The Toda process introduces instantaneous correlation between the stationary and the integrated components of X_{it} through the parameters θ_j, $j = 1, 2, 3$, in the covariance matrix of the innovations ε_{it}. The performance of the test has been investigated both when correlation is present and absent.

If $\psi_a = \psi_b = 1$, then the true cointegrating rank of X_{it} is zero. Since there are no stationary components, $\theta_j = 0$, $\forall j$, and thus each component of X_{it} follows a random walk:

$$X_{it} = X_{i,t-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim N\left(0, I_3\right), \quad \forall i = 1, \ldots, N.$$

If $|\psi_a| < 1$ and $\psi_b = 1$, then the true cointegrating rank of X_{it} is one as it consists of one stationary and two non-stationary components. Allowing for $\theta_1, \theta_2 \neq 0$ introduces correlation between the stationary and non-stationary components.

When both $|\psi_a| < 1$ and $|\psi_b| < 1$, the process for X_{it} has cointegrating rank two. In this case setting $\theta_2, \theta_3 \neq 0$ results in correlation between the single non-stationary and the two stationary components.

Sample sizes of $T - 1 \in \{25, 50, 100\}$ and $N \in \{10, 25, 50, 100\}$ are generated with the initial values of X_{it} set to zero. The large-T behaviour of the test is analysed also for $T - 1 \in \{200, 500\}$ and $N \in \{10, 25, 50\}$.

For the parameters ψ_a and ψ_b only a subset of the parameter values used in the simulation studies in Saikkonen and Lütkepohl (2000) and Örsal and Droge (2012) is selected, namely $\psi_a, \psi_b \in \{1, 0.95, 0.7\}$. For a system with true cointegrating rank zero we set $\psi_a = \psi_b = 1$ and $\theta_j = 0$. A system with cointegrating rank one is simulated with $(\psi_a, \psi_b) \in \{(0.7, 1), (0.95, 1)\}$, each with the following two correlation structures of the errors: $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$ and $(\theta_1, \theta_2, \theta_3) = (0.8, 0.3, 0)$. The combinations $(\psi_a, \psi_b) \in \{(0.7, 0.7), (0.95, 0.7)\}$, each with $(\theta_1, \theta_2, \theta_3) \in \{(0, 0, 0), (0, 0.8, 0.3)\}$, are considered for simulating systems with cointegrating rank two. The linear trend term parameters μ_{0i} and μ_{1i} are set to zero for all i, since they do not affect the results (see also Saikkonen and Lütkepohl (2000), Lütkepohl et al. (2001) and Trenkler (2002)). The number of factors is set to $k = 2$ with $\sigma_F^2 = 1$. In the case of $I(0)$ factors the matrix B is given by ρI_2 for $\rho = 0.9$, and $B = I_2$ when the factors are $I(1)$. The factor loadings are generated as independent uniformly distributed random variables of the corresponding dimension: $\Lambda_i \sim U[-1, 3]$. Prior to extraction of the factors each series is standardised to have zero mean and unit variance.

The simulations are performed in GAUSS 13. The number of replications is 1000, which implies that the standard error of an estimate of the type I error at the 5% significance level is 0.007. The size of the test can be calculated from the reported results as the sum of the proportions for ranks higher than the true rank. Hence, in this setting a test is considered to be undersized if the proportion of rejections of the true cointegrating rank in favour of higher ranks is lower than 0.036, and oversized, if this proportion exceeds 0.074. The (size-unadjusted) power is computed as the sum of the proportions for ranks higher than the hypothesized rank.

4.2 Simulation results

Only results for the experiments considering $I(1)$ factors are reported. The results involving near non-stationary factors are qualitatively the same and are omitted for brevity. The usual 5% nominal size applies in all cases. In all tables PSL_{def}^{J} denotes the Panel SL test applied to the defactored data, where the selector matrix for the stochastic trends $\beta_{i,\perp}$ is estimated using Johansen's approach. Similarly, PSL_{def}^{SW} stands for the Panel SL test applied to the defactored data with $\beta_{i,\perp}$ estimated using the principal components approach of Stock and Watson (1988). PSL_{ind} denotes the Panel SL test applied to the cross-sectionally independent data (i.e. no common components are added to the process X_{it}), which serves as a benchmark for comparison.

Table 2 presents the properties of the test when the true cointegrating rank is zero. The size of the test applied to the cross-sectionally independent data fluctuates around the desired 5% level for all N and T where $T > N$, except when $T = 25$, $N = 10$. When $T < N$ the test becomes oversized, as a result of the condition $N/T \to 0$ not being fulfilled. The results for the defactored data are very similar regardless whether $\hat{\beta}_{\perp}^{\mathrm{J}}$ or $\hat{\beta}_{\perp}^{\mathrm{SW}}$ has been used to select the stochastic trends and resemble those for the cross-sectionally independent data.

The properties of the test when the true cointegrating rank is one with $\psi_a = 0.7$ and $\psi_b = 1$ are presented in Tables 3 and 4. When $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$ and there is no cross-sectional dependence (see Table 3), the Panel SL test has low power only when $T = 25$ and $N < 50$,

Table 2: Chosen rank, proportions, true cointegrating rank $r = 0$

		PSL^{J}_{def}				PSL^{SW}_{def}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.898	0.102	0	0	0.898	0.102	0	0	0.900	0.099	0.001	0
	25	0.827	0.173	0	0	0.827	0.173	0	0	0.857	0.143	0	0
	50	0.784	0.216	0	0	0.784	0.216	0	0	0.803	0.197	0	0
	100	0.710	0.290	0	0	0.710	0.290	0	0	0.693	0.307	0	0
50	10	0.938	0.062	0	0	0.938	0.061	0.001	0	0.936	0.064	0	0
	25	0.922	0.078	0	0	0.922	0.078	0	0	0.919	0.081	0	0
	50	0.916	0.084	0	0	0.916	0.084	0	0	0.920	0.080	0	0
	100	0.866	0.134	0	0	0.866	0.134	0	0	0.886	0.114	0	0
100	10	0.936	0.064	0	0	0.936	0.064	0	0	0.946	0.054	0	0
	25	0.933	0.067	0	0	0.933	0.067	0	0	0.935	0.065	0	0
	50	0.930	0.070	0	0	0.930	0.070	0	0	0.924	0.076	0	0
	100	0.909	0.091	0	0	0.909	0.091	0	0	0.917	0.083	0	0
200	10	0.936	0.064	0	0	0.936	0.064	0	0	0.943	0.057	0	0
	25	0.943	0.057	0	0	0.943	0.057	0	0	0.942	0.058	0	0
	50	0.945	0.055	0	0	0.945	0.055	0	0	0.952	0.048	0	0
500	10	0.946	0.054	0	0	0.946	0.054	0	0	0.945	0.055	0	0
	25	0.947	0.053	0	0	0.947	0.053	0	0	0.949	0.051	0	0
	50	0.936	0.064	0	0	0.936	0.064	0	0	0.935	0.065	0	0

but the probability of correct rank selection increases with N for T fixed. For $T \geq 50$ its power against the null of no cointegration is about 95%. For the defactored data the results are again qualitatively the same for PSL^{J}_{def} and PSL^{SW}_{def}, but their power is lower for small values of N and T compared to the benchmark. The power nevertheless increases to the 95% level for $T, N \geq 50$. The Panel SL test is undersized for small T in all three cases, and the size approaches 5% from below as T increases.

When correlation is introduced through $(\theta_1, \theta_2, \theta_3) = (0.8, 0.3, 0)$ (see Table 4), the size and power in the case of cross-sectionally independent data are excellent even for T and N as small as 25. However, the properties of the Panel SL test applied to the defactored data differ dramatically between PSL^{J}_{def} and PSL^{SW}_{def}. The high correlation coefficient $\theta_1 = 0.8$ between the $I(0)$ component and one of the unit-root processes in X_{it} causes the principal components estimator of Stock and Watson to fail in selecting the stochastic trends correctly. As a result, PSL^{SW}_{def} is severely oversized when testing $H_0 : r = 1$. For T fixed its size increases with N, and for a fixed N it increases with T up to $T = 100$, and subsequently decreases as T grows to 500. The conclusion is that the principal components estimator requires a very large T to cope with the problem. Using it is therefore likely to yield incorrect results in small samples when there exists instantaneous correlation between the innovations of the $I(0)$ and $I(1)$ components. In contrast, the Panel SL test employing Johansen's estimator of $\beta_\perp$ behaves very much like in the benchmark case, maintaining the desired size and power properties for all N and T.

The results for the case of a cointegrating rank one with a near-unit root component are presented in Tables 5 and 6. When $\psi_a = 0.95$, $\psi_b = 1$ and there is no correlation between the innovations ε_{it} (see Table 5), the Panel SL test suffers from low power, requiring at least 200 time observations and 50 cross-sectional units to be able to correctly select the true cointegrating rank in most cases. Nevertheless, the power increases monotonously over N for T fixed in all three settings. As there is no correlation, the results for the defactored data are almost the same for PSL^{J}_{def} and PSL^{SW}_{def}, being also very close to those for the cross-sectionally independent data.

The presence of correlation between the idiosyncratic errors in this setting (see Table 6) results in considerably improved power properties of PSL_{ind}. The results for the defactored data based on PSL^{J}_{def} are also promising and closely mimic those for the cross-sectionally independent data, although PSL^{J}_{def} is not as powerful as PSL_{ind} for small T. However, the correlation again causes the Stock and Watson principal components estimator to fail, whereas the region in which PSL^{SW}_{def} over-rejects $H_0 : r = 1$ is shifted towards higher values of T. The latter may be explained by the Panel SL test being undersized for small T, which offsets the over-rejection caused by the incorrect selection of the stochastic trend(s).

Tables 7 and 8 present the results for systems having true cointegrating rank two with $\psi_a = \psi_b = 0.7$. When there is no correlation between the idiosyncratic errors the Panel SL test selects the correct cointegrating rank for the cross-sectionally independent data in most replications where $T \geq 50$; for $T = 25$ it rather selects cointegrating rank one. It is slightly oversized, though, when testing $H_0 : r = 2$ against $H_1 : r = 3$, with the size fluctuating around the 8% level. The PSL^{SW}_{def} and PSL^{J}_{def} tests on the defactored data are less powerful for small T, but they also achieve more than 90% probability of correct rank selection for $T \geq 100$ and also for $T = 50$ and $N = 100$, with the PSL^{J}_{def} test performing better than PSL^{SW}_{def} for $T = 50$.

When correlation is introduced into the systems, the Panel SL test gains on power (see Table 8), and it already selects the true cointegrating rank in most replications for $T \geq 25$ and $N \geq 50$ when the cross-sectionally independent data is considered. For the defactored data, PSL^{J}_{def} fails for $T = 25$, but performs excellently for $T \geq 50$, $N \geq 25$. Similarly to the case with true rank 1, the test based on PSL^{SW}_{def} is oversized when considering $H_0 : r = 2$ against $H_1 : r = 3$, with the size distortions diminishing for very large T.

The results for systems with cointegrating rank two with one near unit-root component are presented in tables 9 and 10. When $\psi_a = 0.95$, $\psi_b = 0.7$ and $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$, the PSL_{ind} test requires at least $T = 100$, $N = 100$ to select the correct cointegrating rank in most cases. The two tests based on the defactored data are less powerful when $T < 200$, with PSL^{J}_{def} clearly performing better than PSL^{SW}_{def}. In the presence of correlation, the PSL^{SW}_{def} test is again oversized for large T. The PSL^{J}_{def} test, in contrast, behaves very much like the benchmark and selects the correct cointegrating rank in most cases for $T \geq 100$ and $N \geq 25$.

Table 3: Chosen rank, proportions, true rank $r = 1$, $\psi_a = 0.7$, $\psi_b = 1$, $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$

		PSL^{J}_{def}				PSL^{SW}_{def}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.824	0.174	0.002	0	0.824	0.169	0.007	0	0.780	0.215	0.005	0
	25	0.658	0.342	0	0	0.658	0.342	0	0	0.603	0.397	0	0
	50	0.479	0.521	0	0	0.479	0.521	0	0	0.402	0.598	0	0
	100	0.239	0.761	0	0	0.239	0.761	0	0	0.142	0.858	0	0
50	10	0.500	0.499	0.001	0	0.500	0.499	0.001	0	0.331	0.653	0.016	0
	25	0.127	0.872	0.001	0	0.127	0.873	0	0	0.058	0.938	0.004	0
	50	0.006	0.994	0	0	0.006	0.994	0	0	0.001	0.999	0	0
	100	0	1	0	0	0	1	0	0	0	1	0	0
100	10	0.017	0.956	0.027	0	0.017	0.967	0.016	0	0.002	0.956	0.042	0
	25	0	0.979	0.021	0	0	0.981	0.019	0	0	0.956	0.044	0
	50	0	0.981	0.019	0	0	0.983	0.017	0	0	0.954	0.046	0
	100	0	0.989	0.011	0	0	0.992	0.008	0	0	0.963	0.037	0
200	10	0	0.937	0.063	0	0	0.949	0.051	0	0	0.937	0.063	0
	25	0	0.960	0.040	0	0	0.970	0.030	0	0	0.936	0.064	0
	50	0	0.955	0.045	0	0	0.972	0.028	0	0	0.935	0.065	0
500	10	0	0.920	0.079	0.001	0	0.931	0.069	0	0	0.939	0.061	0
	25	0	0.927	0.073	0	0	0.937	0.063	0	0	0.929	0.071	0
	50	0	0.933	0.067	0	0	0.949	0.051	0	0	0.931	0.069	0

Table 4: Chosen rank, proportions, true rank $r = 1$, $\psi_a = 0.7$, $\psi_b = 1$, $(\theta_1, \theta_2, \theta_3) = (0.8, 0.3, 0)$

		PSL^{J}_{def}				PSL^{SW}_{def}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.366	0.619	0.015	0	0.366	0.559	0.075	0	0.046	0.895	0.059	0
	25	0.030	0.966	0.004	0	0.030	0.820	0.150	0	0.001	0.936	0.063	0
	50	0	0.998	0.002	0	0	0.761	0.239	0	0	0.931	0.069	0
	100	0	0.997	0.003	0	0	0.635	0.365	0	0	0.921	0.079	0
50	10	0.005	0.962	0.033	0	0.005	0.700	0.295	0	0	0.931	0.069	0
	25	0	0.976	0.024	0	0	0.413	0.587	0	0	0.940	0.060	0
	50	0	0.985	0.015	0	0	0.140	0.860	0	0	0.942	0.058	0
	100	0	0.994	0.006	0	0	0.013	0.987	0	0	0.942	0.058	0
100	10	0	0.970	0.030	0	0	0.579	0.421	0	0	0.943	0.057	0
	25	0	0.976	0.024	0	0	0.259	0.741	0	0	0.942	0.058	0
	50	0	0.979	0.021	0	0	0.043	0.957	0	0	0.950	0.050	0
	100	0	0.991	0.009	0	0	0	1	0	0	0.952	0.048	0
200	10	0	0.954	0.046	0	0	0.673	0.327	0	0	0.941	0.059	0
	25	0	0.978	0.022	0	0	0.451	0.549	0	0	0.954	0.046	0
	50	0	0.973	0.027	0	0	0.210	0.790	0	0	0.952	0.048	0
500	10	0	0.963	0.037	0	0	0.821	0.179	0	0	0.952	0.048	0
	25	0	0.962	0.038	0	0	0.747	0.253	0	0	0.949	0.051	0
	50	0	0.960	0.040	0	0	0.648	0.352	0	0	0.947	0.053	0

Table 5: Chosen rank, proportions, true rank $r = 1$, $\psi_a = 0.95$, $\psi_b = 1$, $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$

		PSL^J_{def}				PSL^{SW}_{def}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.893	0.107	0	0	0.893	0.104	0.003	0	0.900	0.099	0.001	0
	25	0.831	0.169	0	0	0.831	0.169	0	0	0.853	0.147	0	0
	50	0.771	0.229	0	0	0.771	0.229	0	0	0.791	0.209	0	0
	100	0.711	0.289	0	0	0.711	0.289	0	0	0.689	0.311	0	0
50	10	0.935	0.065	0	0	0.935	0.064	0.001	0	0.932	0.068	0	0
	25	0.913	0.087	0	0	0.913	0.087	0	0	0.892	0.108	0	0
	50	0.889	0.111	0	0	0.889	0.111	0	0	0.886	0.114	0	0
	100	0.820	0.180	0	0	0.820	0.180	0	0	0.831	0.169	0	0
100	10	0.909	0.091	0	0	0.909	0.090	0.001	0	0.911	0.089	0	0
	25	0.861	0.139	0	0	0.861	0.139	0	0	0.855	0.145	0	0
	50	0.810	0.190	0	0	0.810	0.190	0	0	0.798	0.202	0	0
	100	0.696	0.304	0	0	0.696	0.304	0	0	0.694	0.306	0	0
200	10	0.737	0.263	0	0	0.737	0.263	0	0	0.685	0.314	0.001	0
	25	0.510	0.490	0	0	0.510	0.490	0	0	0.441	0.559	0	0
	50	0.199	0.801	0	0	0.199	0.801	0	0	0.183	0.817	0	0
500	10	0.053	0.924	0.023	0	0.053	0.935	0.012	0	0.004	0.977	0.019	0
	25	0	0.983	0.017	0	0	0.993	0.007	0	0	0.996	0.004	0
	50	0	0.990	0.010	0	0	0.996	0.004	0	0	0.997	0.003	0

Table 6: Chosen rank, proportions, true rank $r = 1$, $\psi_a = 0.95$, $\psi_b = 1$, $(\theta_1, \theta_2, \theta_3) = (0.8, 0.3, 0)$

		PSL^J_{def}				PSL^{SW}_{def}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.887	0.111	0.002	0	0.887	0.100	0.013	0	0.871	0.129	0	0
	25	0.826	0.174	0	0	0.826	0.169	0.005	0	0.789	0.211	0	0
	50	0.714	0.286	0	0	0.714	0.282	0.004	0	0.697	0.303	0	0
	100	0.583	0.417	0	0	0.583	0.416	0.001	0	0.521	0.479	0	0
50	10	0.867	0.132	0.001	0	0.867	0.126	0.007	0	0.806	0.194	0	0
	25	0.759	0.240	0.001	0	0.759	0.233	0.008	0	0.669	0.331	0	0
	50	0.570	0.430	0	0	0.570	0.425	0.005	0	0.499	0.501	0	0
	100	0.315	0.685	0	0	0.315	0.683	0.002	0	0.229	0.771	0	0
100	10	0.585	0.408	0.007	0	0.585	0.394	0.021	0	0.346	0.654	0	0
	25	0.170	0.828	0.002	0	0.170	0.791	0.039	0	0.058	0.942	0	0
	50	0.013	0.987	0	0	0.013	0.954	0.033	0	0.004	0.996	0	0
	100	0	1	0	0	0	0.971	0.029	0	0	1	0	0
200	10	0.047	0.931	0.022	0	0.047	0.795	0.158	0	0	0.989	0.011	0
	25	0	0.994	0.006	0	0	0.680	0.320	0	0	1	0	0
	50	0	0.988	0.012	0	0	0.471	0.529	0	0	1	0	0
500	10	0	0.976	0.024	0	0	0.644	0.356	0	0	0.992	0.008	0
	25	0	0.981	0.019	0	0	0.367	0.633	0	0	0.999	0.001	0
	50	0	0.984	0.016	0	0	0.109	0.891	0	0	1	0	0

Table 7: Chosen rank, proportions, true rank $r = 2$, $\psi_a = \psi_b = 0.7$, $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$

		PSL^{J}_{def}				PSL^{SW}_{def}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.665	0.327	0.008	0	0.665	0.325	0.010	0	0.561	0.411	0.028	0
	25	0.381	0.619	0	0	0.381	0.614	0.005	0	0.266	0.702	0.032	0
	50	0.154	0.846	0	0	0.154	0.844	0.002	0	0.066	0.909	0.025	0
	100	0.024	0.976	0	0	0.024	0.976	0	0	0	0.973	0.027	0
50	10	0.050	0.702	0.238	0.010	0.050	0.742	0.208	0	0.007	0.401	0.557	0.035
	25	0	0.413	0.580	0.007	0	0.531	0.469	0	0	0.089	0.887	0.024
	50	0	0.138	0.859	0.003	0	0.243	0.757	0	0	0.003	0.974	0.023
	100	0	0.020	0.980	0	0	0.062	0.938	0	0	0	0.984	0.016
100	10	0	0.009	0.925	0.066	0	0.029	0.954	0.017	0	0	0.911	0.089
	25	0	0	0.965	0.035	0	0	0.996	0.004	0	0	0.924	0.076
	50	0	0	0.963	0.037	0	0	0.999	0.001	0	0	0.902	0.098
	100	0	0	0.986	0.014	0	0	1	0	0	0	0.891	0.109
200	10	0	0	0.921	0.079	0	0	0.944	0.056	0	0	0.922	0.078
	25	0	0	0.940	0.060	0	0	0.966	0.034	0	0	0.916	0.084
	50	0	0	0.958	0.042	0	0	0.984	0.016	0	0	0.920	0.080
500	10	0	0	0.901	0.099	0	0	0.924	0.074	0	0	0.928	0.072
	25	0	0	0.927	0.073	0	0	0.942	0.058	0	0	0.934	0.066
	50	0	0	0.934	0.066	0	0	0.951	0.049	0	0	0.928	0.072

Table 8: Chosen rank, proportions, true rank $r = 2$, $\psi_a = \psi_b = 0.7$, $(\theta_1, \theta_2, \theta_3) = (0, 0.8, 0.3)$

		PSL^{J}_{def}				PSL^{SW}_{def}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.204	0.749	0.047	0	0.204	0.604	0.189	0.003	0.018	0.731	0.248	0.003
	25	0.006	0.931	0.063	0	0.006	0.526	0.468	0	0	0.526	0.474	0
	50	0	0.859	0.141	0	0	0.274	0.726	0	0	0.249	0.751	0
	100	0	0.781	0.219	0	0	0.082	0.918	0	0	0.059	0.941	0
50	10	0.001	0.380	0.601	0.018	0.001	0.096	0.868	0.035	0	0.065	0.929	0.006
	25	0	0.036	0.951	0.013	0	0	0.950	0.050	0	0.003	0.996	0.001
	50	0	0	0.995	0.005	0	0	0.954	0.046	0	0	0.999	0.001
	100	0	0	0.997	0.003	0	0	0.934	0.066	0	0	1	0
100	10	0	0.002	0.962	0.036	0	0	0.878	0.122	0	0	0.977	0.023
	25	0	0	0.976	0.024	0	0	0.763	0.237	0	0	0.989	0.011
	50	0	0	0.973	0.027	0	0	0.640	0.360	0	0	0.990	0.010
	100	0	0	0.989	0.011	0	0	0.458	0.542	0	0	0.998	0.002
200	10	0	0	0.969	0.031	0	0	0.856	0.144	0	0	0.968	0.032
	25	0	0	0.969	0.031	0	0	0.762	0.238	0	0	0.984	0.016
	50	0	0	0.971	0.029	0	0	0.641	0.359	0	0	0.982	0.018
500	10	0	0	0.960	0.040	0	0	0.882	0.118	0	0	0.952	0.048
	25	0	0	0.955	0.045	0	0	0.832	0.168	0	0	0.957	0.043
	50	0	0	0.958	0.042	0	0	0.778	0.222	0	0	0.962	0.038

Table 9: Chosen rank, proportions, true rank $r = 2$, $\psi_a = 0.95$, $\psi_b = 0.7$, $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$

		PSL_{def}^{J}				PSL_{def}^{SW}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.809	0.190	0.001	0	0.809	0.185	0.006	0	0.761	0.236	0.003	0
	25	0.649	0.351	0	0	0.649	0.351	0	0	0.604	0.394	0.002	0
	50	0.474	0.526	0	0	0.474	0.526	0	0	0.386	0.614	0	0
	100	0.281	0.719	0	0	0.281	0.719	0	0	0.133	0.867	0	0
50	10	0.496	0.500	0.004	0	0.496	0.498	0.006	0	0.311	0.667	0.022	0
	25	0.112	0.885	0.003	0	0.112	0.884	0.004	0	0.044	0.944	0.012	0
	50	0.004	0.996	0	0	0.004	0.995	0.001	0	0	0.990	0.010	0
	100	0	0.999	0.001	0	0	1	0	0	0	0.992	0.008	0
100	10	0.008	0.903	0.088	0.001	0.008	0.928	0.064	0	0	0.853	0.146	0.001
	25	0	0.860	0.140	0	0	0.918	0.082	0	0	0.719	0.281	0
	50	0	0.759	0.241	0	0	0.869	0.131	0	0	0.555	0.445	0
	100	0	0.596	0.404	0	0	0.799	0.201	0	0	0.291	0.709	0
200	10	0	0.439	0.548	0.013	0	0.514	0.482	0.004	0	0.335	0.654	0.011
	25	0	0.100	0.890	0.010	0	0.140	0.860	0	0	0.048	0.951	0.001
	50	0	0.006	0.994	0	0	0.016	0.984	0	0	0.004	0.996	0
500	10	0	0	0.931	0.069	0	0.001	0.970	0.029	0	0	0.948	0.052
	25	0	0	0.949	0.051	0	0	0.982	0.018	0	0	0.967	0.033
	50	0	0	0.958	0.042	0	0	0.992	0.008	0	0	0.958	0.042

Table 10: Chosen rank, proportions, true rank $r = 2$, $\psi_a = 0.95$, $\psi_b = 0.7$, $(\theta_1, \theta_2, \theta_3) = (0, 0.8, 0.3)$

		PSL_{def}^{J}				PSL_{def}^{SW}				PSL_{ind}			
T	N	0	1	2	3	0	1	2	3	0	1	2	3
25	10	0.719	0.277	0.004	0	0.719	0.228	0.053	0	0.571	0.416	0.013	0
	25	0.461	0.533	0.006	0	0.461	0.438	0.101	0	0.282	0.712	0.006	0
	50	0.210	0.788	0.002	0	0.210	0.644	0.146	0	0.076	0.921	0.003	0
	100	0.045	0.953	0.002	0	0.045	0.735	0.220	0	0.003	0.995	0.002	0
50	10	0.240	0.725	0.034	0.001	0.240	0.499	0.259	0.002	0.033	0.892	0.075	0
	25	0.003	0.890	0.107	0	0.003	0.311	0.686	0	0	0.907	0.093	0
	50	0	0.815	0.185	0	0	0.052	0.948	0	0	0.879	0.121	0
	100	0	0.633	0.367	0	0	0	1	0	0	0.801	0.199	0
100	10	0.001	0.592	0.398	0.009	0.001	0.185	0.796	0.018	0	0.415	0.585	0
	25	0	0.168	0.827	0.005	0	0.005	0.982	0.013	0	0.105	0.895	0
	50	0	0.018	0.981	0.001	0	0	0.986	0.014	0	0.007	0.993	0
	100	0	0	0.998	0.002	0	0	0.987	0.013	0	0	1	0
200	10	0	0.019	0.962	0.019	0	0.008	0.892	0.100	0	0.001	0.997	0.002
	25	0	0	0.991	0.009	0	0	0.804	0.196	0	0	1	0
	50	0	0	0.993	0.007	0	0	0.650	0.350	0	0	1	0
500	10	0	0	0.967	0.033	0	0	0.784	0.216	0	0	1	0
	25	0	0	0.977	0.023	0	0	0.586	0.414	0	0	0.999	0.001
	50	0	0	0.983	0.017	0	0	0.353	0.647	0	0	1	0

5 Conclusion

In this paper we propose an extension of the Panel SL cointegration rank test by Örsal and Droge (2012) allowing for cross-sectional dependence. The dependence is modelled by unobserved common factors which may be stationary or integrated or a combination of both and which are allowed to affect all variables through heterogeneous loadings. The factors and their loadings are estimated by principal components from the first differenced and demeaned observations as proposed by Bai and Ng (2004). In this way the integrating and cointegrating properties of the unobserved common and idiosyncratic components can be tested independently of each other.

The null hypothesis of no cointegration among the idiosyncratic components is then tested directly by the Panel SL test applied to the defactored data. Testing the null hypothesis of cointegrating rank greater than zero is performed by applying the Panel SL test for no cointegration to estimates of the idiosyncratic stochastic trends. The latter are extracted through a consistent estimate of the orthogonal complement of the space spanned by the cointegrating relations. Two such estimators computed from the defactored data have been considered: the principal components estimator of Stock and Watson (1988) and the estimator of Johansen (1995).

A Monte Carlo simulation study demonstrates that the proposed rank testing procedure which selects the stochastic trends by the Johansen's estimator (the PSL^{J}_{def} test) preserves the properties of the Panel SL test for independent data (PSL_{ind}) in all experimental settings considered. The PSL^{J}_{def} test has the correct size when testing the null hypothesis of no cointegration for $T \geq N$, and it is undersized when testing for higher cointegrating ranks, approaching the correct size from below as T grows large. Although it is less powerful than PSL_{ind} in some cases for short time series, PSL^{J}_{def} continues to offer significant power gains as the number of cross-sections increases. In contrast, the Panel SL rank testing procedure employing the principal components estimator of Stock and Watson (PSL^{SW}_{def}) turns out to perform unsatisfactorily in the presence of correlation between the innovations to the stationary and non-stationary components of the idiosyncratic processes. In those cases the PSL^{SW}_{def} test for a cointegrating rank greater than zero becomes oversized, with the size distortions diminishing only as T grows very large. We would thus recommend the PSL^{J}_{def} test for use in empirical research.

Several directions for further development of the Panel SL test can be outlined. Other ways for controlling for the cross-sectional dependence can be explored, for example approximating the common factors by the cross-sectional averages of the observed data following Pesaran (2006). The latter approach is expected to be more suitable for small values of N and T when the method of principal components may yield imprecise estimates of the common factors and their loadings. Extensions to the case of weak cross-sectional dependence, for example spatial type of dependence, can also be considered.

Appendix

In the following we shall show that testing the null hypothesis of no cointegration by the Panel SL test using the defactored data yields asymptotically equivalent results as using the cross-sectionally independent idiosyncratic components. We first outline some general considerations and prove auxiliary lemmas necessary for the proof of Theorem 3.1.

Recall that the cross-sectionally independent processes Y_{it} and the defactored observed processes Y_{it}^* are defined as

$$\begin{aligned} Y_{it} &= \mu_{0i} + \mu_{1i}t + X_{it}, \\ Y_{it}^* &= \mu_{0i} + \mu_{1i}t + X_{it} + \Lambda_i' F_t - \hat{\Lambda}_i' \hat{F}_t, \end{aligned}$$

where $\hat{F}_t = \sum_{s=2}^t \hat{f}_s$ and $f_t = \Delta F_t - \overline{\Delta F}$.

Bai and Ng (2004) have shown that

$$\hat{F}_t = H\left(F_t - F_1 - \frac{F_T - F_1}{T-1}(t-1)\right) + V_t, \tag{A.1}$$

where $V_t = \sum_{s=2}^t v_s = \sum_{s=2}^t \left(\hat{f}_s - H f_s\right)$ and $\|V_t\| = O_p\left(\sqrt{\frac{T}{N}}\right)$. H is a full rank $(k \times k)$ matrix defined as $H = V_{NT}^{-1}\left(\hat{f}' f/(T-1)\right)(\Lambda'\Lambda/(Nm))$, where V_{NT} is a diagonal matrix with the k largest eigenvalues of $yy'/\left((T-1)Nm\right)$ in decreasing order on the main diagonal. Bai (2003) has shown that $\|H\| = O_p(1)$.

We make further use of the notation of Bai and Ng (2004) and let $D_i = \hat{\Lambda}_i - \left(H^{-1}\right)' \Lambda_i$. Note that in the current formulation of the model Λ_i and D_i are $(k \times m)$ matrices, as opposed to $(k \times 1)$ vectors in Bai and Ng (2004). Lemma 1 of Bai and Ng (2004) establishes that for the l-th column of D_i it holds that

$$\|D_i(l)\| = O_p\left(\frac{1}{\min(\sqrt{T}, N)}\right) \quad \text{for each } i = 1, \ldots, N. \tag{A.2}$$

Therefore for the matrix D_i it holds that

$$\|D_i\| \leq \sqrt{\sum_{l=1}^m \|D_i(l)\|^2} \leq \sqrt{m} O_p\left(\frac{1}{\min(\sqrt{T}, N)}\right) \leq \sqrt{m} O_p\left(C_{NT}^{-1}\right). \tag{A.3}$$

Now, substituting $\hat{F}_t$ with the expression in (A.1), the difference between the defactored and the cross-sectionally independent process can be decomposed into a linear time trend and a stochastic trend:

$$\begin{aligned}
Y_{it}^{*} - Y_{it} &= \Lambda_i' F_t - \hat{\Lambda}_i' \hat{F}_t \\
&= \Lambda_i' H^{-1} H F_t - \hat{\Lambda}_i' \left[\left(H F_t - H F_1 - H \frac{F_T - F_1}{T-1}(t-1) \right) + V_t \right] \\
&= \left(\Lambda_i' H^{-1} - \hat{\Lambda}_i' \right) H F_t + \hat{\Lambda}_i' H F_1 + \hat{\Lambda}_i' H \frac{F_T - F_1}{T-1}(t-1) - \hat{\Lambda}_i' V_t \\
&= \left(\Lambda_i' H^{-1} - \hat{\Lambda}_i' \right) H F_t + \hat{\Lambda}_i' H F_1 - \Lambda_i' H^{-1} H F_1 + \Lambda_i' F_1 \\
&\quad + \hat{\Lambda}_i' H \frac{F_T - F_1}{T-1}(t-1) - \Lambda_i' H^{-1} H \frac{F_T - F_1}{T-1}(t-1) \\
&\quad + \Lambda_i' \frac{F_T - F_1}{T-1}(t-1) - \hat{\Lambda}_i' V_t + \Lambda_i' H^{-1} V_t - \Lambda_i' H^{-1} V_t \\
&= \left(\Lambda_i' H^{-1} - \hat{\Lambda}_i' \right) H F_t + (\hat{\Lambda}_i' - \Lambda_i' H^{-1}) H F_1 + \Lambda_i' F_1 \\
&\quad + \left(\hat{\Lambda}_i' - \Lambda_i' H^{-1} \right) H \frac{F_T - F_1}{T-1}(t-1) + \Lambda_i' \frac{F_T - F_1}{T-1}(t-1) \\
&\quad - \left(\hat{\Lambda}_i' - \Lambda_i' H^{-1} \right) V_t - \Lambda_i' H^{-1} V_t \\
&= D_i' H F_1 + \Lambda_i' F_1 + D_i' H \frac{F_T - F_1}{T-1}(t-1) + \Lambda_i' \frac{F_T - F_1}{T-1}(t-1) \qquad \text{(A.4)} \\
&\quad - D_i' H F_t - D_i' V_t - \Lambda_i' H^{-1} V_t, \qquad \text{(A.5)}
\end{aligned}$$

where the time trend terms are grouped together in (A.4) and the stochastic trend terms are in (A.5). The time trend in (A.4) can be added to the existing time trend $\mu_{0i} + \mu_{1i} t$ of the process. Therefore no prior detrending of the factor estimates $\hat{F}_t$ is required, since both the original time trend and the time trend arising from the defactoring procedure will be simultaneously estimated by the GLS procedure in the next step.

Defining the new combined trend parameters of the defactored process as

$$\begin{aligned}
\mu_{0i}^{+} &= \mu_{0i} + D_i' H F_1 + \Lambda_i' F_1 - D_i' H \frac{F_T - F_1}{T-1} - \Lambda_i' \frac{F_T - F_1}{T-1} \quad \text{and} \\
\mu_{1i}^{+} &= \mu_{1i} + D_i' H \frac{F_T - F_1}{T-1} + \Lambda_i' \frac{F_T - F_1}{T-1},
\end{aligned}$$

the process Y_{it}^{*} can be written as

$$Y_{it}^{*} = \mu_{0i}^{+} + \mu_{1i}^{+} t + X_{it} - D_i' H F_t - D_i' V_t - \Lambda_i' H^{-1} V_t. \qquad \text{(A.6)}$$

Introducing the shorthand notation for the stochastic trend term in (A.5)

$$\eta_{it} = D_i' H F_t + D_i' V_t + \Lambda_i' H^{-1} V_t,$$

we obtain

$$Y_{it}^{*} = \mu_{0i}^{+} + \mu_{1i}^{+} t + X_{it} - \eta_{it}$$

and we note that $\|\eta_{it}\| = O_p\left(\sqrt{\frac{T}{N}}\right) + O_p(1)$ and $\|\Delta \eta_{it}\| = o_P(1)$.

It is easily seen that the estimate of the idiosyncratic component $\hat{X}_{it} = \sum_{s=2}^{t} \left(y_{is} - \hat{\Lambda}_i' \hat{f}_s \right)$ differs from Y_{it}^* only in the formulation of the constant and the time trend term:

$$\begin{aligned}\hat{X}_{it} &= X_{it} + \left(D_i' H F_1 - X_{i1}\right) + \left(D_i' H \frac{F_T - F_1}{T-1} - \frac{X_{iT} - X_{i1}}{T-1} \right)(t-1) \\ &\quad - D_i' H F_t - D_i' V_t - \Lambda_i' H^{-1} V_t, \\ &= X_{it} + \mu_{0i}^{++} + \mu_{1i}^{++} t - \eta_{it}.\end{aligned}$$

In other words, while Y_{it}^* contains both the original time trend and a time trend arising from the defactoring, $\hat{X}_{it}$ contains only deterministic terms arising from recovering the idiosyncratic and common components, with the time trend parameter diminishing to zero as $O_p\left(T^{-1/2}\right)$ and the intercept term being $O_p(1)$.

From these representations it is clear that $\beta_i' Y_{it}^*$ and $\beta_i' \hat{X}_{it}$ will not be $I(0)$ in the general case, because β_i is not necessarily a cointegrating vector for the stochastic trend term η_{it}. Under the null hypothesis $H_0 : r_i = 0, \forall i$, however, $\beta_i = \alpha_i = 0$ and thus the stochastic trend arising from the defactoring disappears from the cointegrating relation in the VECM representation of Y_{it}^*:

$$\Delta Y_{it}^* = \nu_i^+ + \alpha_i \left(\beta_i' Y_{i,t-1}^* - \beta_i' \eta_{i,t-1} - \tau_i^+ (t-1) \right) + \sum_{j=1}^{p_i - 1} \Gamma_{ij} \left(\Delta Y_{i,t-j}^* \right) + \varepsilon_{it}^+, \tag{A.7}$$

where $\nu_i^+ = -\Pi_i \mu_{i0}^+ + \left(I_m - \sum_{j=1}^{p_i} \Gamma_{ij} \right) \mu_{1i}^+$ and $\tau_i^+ = \beta_i' \mu_{1i}^+$. The residual term ε_{it}^+ stands for $\varepsilon_{it}^+ = \varepsilon_{it} + \Delta \eta_{it} - \sum_{j=1}^{p_i - 1} \Gamma_{ij} \Delta \eta_{i,t-j} = \varepsilon_{it} + o_P(1)$, therefore $\frac{1}{T} \sum_{t=1}^{T} \left(\varepsilon_{it}^+ \varepsilon_{it}^{+\prime} \right) \xrightarrow{p} \Omega_i$ as $T \to \infty$ for each i.

Applying the GLS trend-adjustment of Saikkonen and Lütkepohl (2000) to Y_{it} and Y_{it}^* under $H_0 : r_i = 0$ for all i yields

$$\begin{aligned}\tilde{X}_{it} &= Y_{it} - \tilde{\mu}_{0i} - \tilde{\mu}_{1i} t \\ &= X_{it} + (\mu_{0i} - \tilde{\mu}_{0i}) + (\mu_{1i} - \tilde{\mu}_{1i}) t, \end{aligned} \tag{A.8}$$

$$\begin{aligned}\tilde{X}_{it}^* &= Y_{it}^* - \tilde{\mu}_{0i}^+ - \tilde{\mu}_{1i}^+ t \\ &= X_{it} + (\mu_{0i}^+ - \tilde{\mu}_{0i}^+) + (\mu_{1i}^+ - \tilde{\mu}_{1i}^+) t - D_i' H F_t - D_i' V_t - \Lambda_i' H^{-1} V_t,\end{aligned} \tag{A.9}$$

where

$$\| \mu_{0i}^+ - \tilde{\mu}_{0i}^+ \| = \| \mu_{0i} - \tilde{\mu}_{0i} \| = O_p(1), \tag{A.10}$$

$$\| \mu_{1i}^+ - \tilde{\mu}_{1i}^+ \| = \| \mu_{1i} - \tilde{\mu}_{1i} \| = O_p\left(\frac{1}{\sqrt{T}} \right) \tag{A.11}$$

by Theorem 1 of Saikkonen and Lütkepohl (2000). The GLS-detrending under the null hypothesis of no cointegration leaves $\hat{X}_{it}$ unchanged and also numerically equal to $\tilde{X}_{it}^*$. We can therefore without loss of generality assume that

$$\tilde{X}_{it}^* = \hat{X}_{it} = \tilde{X}_{it} - D_i' H F_t - D_i' V_t - \Lambda_i' H^{-1} V_t = \tilde{X}_{it} - \eta_{it}. \tag{A.12}$$

We shall now re-state a result of Bai and Ng (2004) in terms of the processes $\tilde{X}_{it}^*$ and $\tilde{X}_{it}$. In particular, the statement of Lemma G.1 from the aforementioned article holds for each element $\tilde{X}_{it}^*(l)$ and $\tilde{X}_{it}(l)$, $l = 1, \ldots, m$, of the vector processes $\tilde{X}_{it}^*$ and $\tilde{X}_{it}$, respectively.

Lemma A.1. *For each cross-section $i = 1, \ldots, N$ and for each pair of elements $\tilde{X}_{it}(l)$ and $\tilde{X}_{it}^*(l)$ of the vector processes $\tilde{X}_{it}^*$ and $\tilde{X}_{it}$, $l = 1, \ldots, m$, it holds that:*

(i) $\frac{1}{\sqrt{T}}\tilde{X}_{it}^*(l) = \frac{1}{\sqrt{T}}\tilde{X}_{it}(l) + O_p\left(C_{NT}^{-1}\right)$,

(ii) $\frac{1}{T^2}\sum_{t=2}^T \tilde{X}_{it}^*(l)^2 = \frac{1}{T^2}\sum_{t=2}^T \tilde{X}_{it}(l)^2 + O_p\left(C_{NT}^{-1}\right)$,

(iii) $\frac{1}{T}\sum_{t=2}^T \Delta\tilde{X}_{it}^*(l)^2 = \frac{1}{T}\sum_{t=2}^T \Delta\tilde{X}_{it}(l)^2 + O_p\left(C_{NT}^{-1}\right)$,

(iv) $\frac{1}{T}\sum_{t=2}^T \tilde{X}_{i,t-1}^*(l)\Delta\tilde{X}_{it}^*(l) = \frac{1}{T}\sum_{t=2}^T \tilde{X}_{i,t-1}(l)\Delta\tilde{X}_{it}(l) + O_p\left(C_{NT}^{-1}\right)$,

(v) $\frac{1}{T}\sum_{t=2}^T \left(\Delta\tilde{X}_{i,t}^*(l) - \Delta\tilde{X}_{it}(l)\right)^2 = O_p\left(C_{NT}^{-2}\right)$.

Proof. The proofs of (i) – (iv) follow the same lines as those of Lemma G.1 in the Appendix of Bai and Ng (2004) and are thus omitted. We prove (v):

$$\begin{aligned}
&\frac{1}{T}\sum_{t=2}^T \left(\Delta\tilde{X}_{i,t}^*(l) - \Delta\tilde{X}_{it}(l)\right)^2 \\
&= \frac{1}{T}\sum_{t=2}^T \left(\Delta\tilde{X}_{i,t}(l) - D_i(l)'Hf_t - D_i(l)'v_t - \Lambda_i(l)'H^{-1}v_t - \Delta\tilde{X}_{i,t}(l)\right)^2 \\
&\leq \|D_i(l)'\|^2\|H\|^2\frac{1}{T}\sum_{t=2}^T\|f_t\|^2 + \|D_i(l)'\|^2\frac{1}{T}\sum_{t=2}^T\|v_t\|^2 \\
&\quad + \|\Lambda_i(l)'H^{-1}\|^2\frac{1}{T}\sum_{t=2}^T\|v_t\|^2 \\
&\leq O_p\left(C_{NT}^{-2}\right) + O_p\left(C_{NT}^{-2}\right)O_p\left(C_{NT}^{-2}\right) + O_p\left(C_{NT}^{-2}\right) = O_p\left(C_{NT}^{-2}\right),
\end{aligned}$$

since $\|D_i(l)\|^2 \leq O_p\left(C_{NT}^{-2}\right)$ by (A.2), $\|\Lambda_i(l)\| = O_p(1)$ by Assumption 2, $\|H\| = O_p(1)$ by Bai (2003), $\frac{1}{T}\sum_{t=2}^T\|f_t\|^2 = O_p(1)$ since by definition $f_t = \Delta F_t - \overline{\Delta F} \sim I(0)$ and $\frac{1}{T}\sum_{t=2}^T\|v_t\|^2 = O_p\left(C_{NT}^{-2}\right)$ by Lemma 1 (a) of Bai and Ng (2004). □

We further generalize these results for the vector processes as follows.

Lemma A.2. *For each cross-section $i = 1, \ldots, N$ for the vector processes $\tilde{X}_{it}^*$ and $\tilde{X}_{it}$ it holds that:*

(i) $\frac{1}{\sqrt{T}}\tilde{X}_{it}^* = \frac{1}{\sqrt{T}}\tilde{X}_{it} + \sqrt{m}\,O_p\left(C_{NT}^{-1}\right)$,

(ii) $\frac{1}{T^2}\sum_{t=2}^T \tilde{X}_{it}^*\tilde{X}_{it}^{*\prime} = \frac{1}{T^2}\sum_{t=2}^T \tilde{X}_{it}\tilde{X}_{it}' + m\,O_p\left(C_{NT}^{-1}\right)$,

(iii) $\frac{1}{T}\sum_{t=2}^T \Delta\tilde{X}_{it}^*\Delta\tilde{X}_{it}^{*\prime} = \frac{1}{T}\sum_{t=2}^T \Delta\tilde{X}_{it}\Delta\tilde{X}_{it}' + m\,O_p\left(C_{NT}^{-1}\right)$,

(iv) $\frac{1}{T}\sum_{t=2}^{T}\tilde{X}^{*}_{i,t-1}\Delta\tilde{X}_{it}{}^{*\prime} = \frac{1}{T}\sum_{t=2}^{T}\tilde{X}_{i,t-1}\Delta\tilde{X}_{it}' + m\,O_p\left(C_{NT}^{-1}\right)$,

(v) $\frac{1}{T}\sum_{t=2}^{T}\left\|\Delta\tilde{X}^{*}_{i,t} - \Delta\tilde{X}_{it}\right\|^2 = m\,O_p\left(C_{NT}^{-2}\right)$.

Proof. (i) Follows from

$$\begin{aligned}\frac{1}{\sqrt{T}}\left\|\tilde{X}^{*}_{it} - \tilde{X}_{it}\right\| &= \frac{1}{\sqrt{T}}\sqrt{\sum_{l=1}^{m}\left(\tilde{X}^{*}_{it}(l) - \tilde{X}_{it}(l)\right)^2} \\ &\leq \sqrt{m}\max_{1\leq l\leq m}\left(\frac{1}{\sqrt{T}}\left|\tilde{X}^{*}_{it}(l) - \tilde{X}_{it}(l)\right|\right) \\ &\leq \sqrt{m}\,O_p\left(C_{NT}^{-1}\right),\end{aligned}$$

since from Lemma A.1 (i) we have that $\frac{1}{\sqrt{T}}\left|\tilde{X}^{*}_{it}(l) - \tilde{X}_{it}(l)\right| = O_p\left(C_{NT}^{-1}\right)$ for each l, i.

To show (ii) we shall make use of the intermediate result:

$$\frac{1}{T^2}\sum_{t=2}^{T}\tilde{X}^{*}_{it}(l)\tilde{X}^{*}_{it}(j) = \frac{1}{T^2}\sum_{t=2}^{T}\tilde{X}_{it}(l)\tilde{X}_{it}(j) + O_p\left(C_{NT}^{-1}\right) \text{ for } j, l = 1, \ldots, m;\ j \neq l. \tag{A.13}$$

To see this, consider the expression

$$\left|\frac{1}{T^2}\sum_{t=2}^{T}\left(\tilde{X}^{*}_{it}(l)\tilde{X}^{*}_{it}(j) - \tilde{X}_{it}(l)\tilde{X}_{it}(j)\right)\right|,$$

whose order of convergence, after expanding both $\tilde{X}^{*}_{it}(l)$ and $\tilde{X}^{*}_{it}(j)$ as

$$\tilde{X}^{*}_{it}(l) = \tilde{X}_{it}(l) - D_i(l)'HF_t - D_i(l)'V_t - \Lambda_i(l)'H^{-1}V_t,$$

is dominated by the component

$$\begin{aligned}&\left|\frac{1}{T^2}\sum_{t=2}^{T}\left(\tilde{X}_{it}(j)D_i(l)'HF_t - \tilde{X}_{it}(j)D_i(l)'V_t - \tilde{X}_{it}(j)\Lambda_i(l)'H^{-1}V_t\right)\right| \\ &\leq \left(\frac{1}{T^2}\sum_{t=2}^{T}\tilde{X}_{it}(j)^2\right)^{1/2}\|D_i(l)\|\|H\|\left(\frac{1}{T^2}\sum_{t=2}^{T}\|F_t\|^2\right)^{1/2} \\ &\quad + \left(\frac{1}{T^2}\sum_{t=2}^{T}\tilde{X}_{it}(j)^2\right)^{1/2}\|D_i(l)\|\frac{1}{\sqrt{T}}\left(\frac{1}{T}\sum_{t=2}^{T}\|V_t\|^2\right)^{1/2} \\ &\quad + \left(\frac{1}{T^2}\sum_{t=2}^{T}\tilde{X}_{it}(j)^2\right)^{1/2}\|\Lambda_i(l)'H^{-1}\|\frac{1}{\sqrt{T}}\left(\frac{1}{T}\sum_{t=2}^{T}\|V_t\|^2\right)^{1/2} \\ &\leq O_p\left(C_{NT}^{-1}\right) + O_p\left(N^{-1/2}\right)O_p\left(C_{NT}^{-1}\right) + O_p\left(C_{NT}^{-1}\right) = O_p\left(C_{NT}^{-1}\right),\end{aligned}$$

since $\left(\frac{1}{T^2}\sum_{t=2}^{T}\tilde{X}_{it}(j)^2\right)^{1/2} = O_p(1)$ by definition, $\|D_i(l)\| \leq O_p\left(C_{NT}^{-1}\right)$, $\|\Lambda_i(l)'H^{-1}\| = O_p(1)$ by the same argument as in the proof of Lemma A.1 (v) above and $\left(\frac{1}{T}\sum_{t=2}^{T}\|V_t\|^2\right)^{1/2} = O_p\left(\sqrt{\frac{T}{N}}\right)$ as shown by Bai and Ng (2004).

Now, considering the difference in the vector processes, by (A.13) and Lemma A.1 (i) we obtain

$$\begin{aligned}\left\|\frac{1}{T^2}\sum_{t=2}^{T}\left(\tilde{X}_{it}^*\tilde{X}_{it}^{*\prime} - \tilde{X}_{it}\tilde{X}_{it}'\right)\right\|^2 &\leq \sum_{j,l=1}^{m}\left|\frac{1}{T^2}\sum_{t=2}^{T}\left(\tilde{X}_{it}^*(j)\tilde{X}_{it}^*(l) - \tilde{X}_{it}(j)\tilde{X}_{it}(l)\right)\right|^2 \\ &\leq m^2 O_p\left(C_{NT}^{-2}\right),\end{aligned}$$

which yields the desired result.

The proofs of (iii),(iv) and (v) use similar arguments and are omitted for brevity. □

In order to analyse the derivation and the asymptotic behaviour of the LR trace statistics $\mathrm{LR}_{\mathrm{trace}_{iT}}^{\mathrm{SL}^*}(r)$ and $\mathrm{LR}_{\mathrm{trace}_{iT}}^{\mathrm{SL}}(r)$ we shall make use of the notation in Johansen (1995). Let

$$\begin{aligned}Z_{i,0t} &= \Delta\tilde{X}_{it}, \\ Z_{i,1t} &= \tilde{X}_{i,t-1} \text{ and} \\ Z_{i,2t} &= \left(\Delta\tilde{X}_{i,t-1}', \ldots, \Delta\tilde{X}_{i,t-p_i+1}'\right)'.\end{aligned} \tag{A.14}$$

We also introduce the product moment matrices $M_{i,jk}$, $j,k = 0,1,2$ as

$$M_{i,jk} = \frac{1}{T}\sum_{t=1}^{T} Z_{i,jt}Z_{i,kt}',$$

and the cross-product matrices $S_{i,jk}, j,k = 0,1$, which are defined as follows:

$$\begin{aligned}S_{i,00} &= M_{i,00} - M_{i,02}M_{i,22}^{-1}M_{i,20}, \\ S_{i,10} &= M_{i,10} - M_{i,12}M_{i,22}^{-1}M_{i,20}, \\ S_{i,11} &= M_{i,11} - M_{i,12}M_{i,22}^{-1}M_{i,21}.\end{aligned}$$

In the same way we define the matrices $Z_{i,jt}^*$, $M_{i,jk}^*$, $j,k = 0,1,2$ and $S_{i,jk}^*, j,k = 0,1$ in terms of the defactored and detrended process $\tilde{X}_{it}^*$ defined by (A.12).

The individual LR trace statistic based on the cross-sectionally independent and detrended data is defined as

$$\mathrm{LR}_{\mathrm{trace}_{iT}}^{\mathrm{SL}}(r) = -T\sum_{j=r+1}^{m}\ln\left(1 - \hat{\lambda}_{ij}\right),$$

where $\hat{\lambda}_{i1} > \ldots > \hat{\lambda}_{im}$ denote the ordered estimated eigenvalues of the eigenvalue problem defined in Johansen (1995, pp. 90-93):

$$|S_i(\lambda)| = \left|\lambda S_{i,11} - S_{i,10}S_{i,00}^{-1}S_{i,01}\right| = 0. \tag{A.15}$$

The LR trace statistic for the defactored and detrended data $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace}_{iT}}(r)$ is defined in an analogous manner using the starred versions of the cross-product matrices. In the following arguments we shall make use of the following proposition:

Proposition A.1. *If Assumption 1 holds, under the null hypothesis of no cointegration all solutions to the eigenvalue problems* $|S_i(\lambda)| = 0$ *and* $|S^*_i(\lambda)| = 0$ *are* $O_p(T^{-1})$.

Proof. The statement of the proposition follows from the fact that under the null hypothesis $\|S_{i,11}\| = O_p(T)$, while $\|S_{i,01}\| = \|S_{i,00}\| = \|S^{-1}_{i,00}\| = \|S_{i,10}\| = O_p(1)$, which follows from Lemma A.3 of Saikkonen and Lütkepohl (2000). The corresponding results for the starred matrices follow from those above and Lemma A.4. □

The statement of the proposition allows both LR trace statistics for testing for no cointegration – $\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0)$ derived from the cross-sectionally independent data and $\mathrm{LR}^{\mathrm{SL}^*}_{\mathrm{trace_{iT}}}(0)$ based on the defactored data – to be approximated by the first terms of the corresponding Taylor expansion:

$$\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0) = -T\sum_{j=1}^{m} \ln\left(1-\hat{\lambda}_{ij}\right) = T\sum_{j=r+1}^{m} \hat{\lambda}_{ij} + O_p(T^{-1}),$$
$$\mathrm{LR}^{\mathrm{SL}^*}_{\mathrm{trace_{iT}}}(0) = -T\sum_{j=1}^{m} \ln\left(1-\hat{\lambda}^*_{ij}\right) = T\sum_{j=r+1}^{m} \hat{\lambda}^*_{ij} + O_p(T^{-1}).$$

Therefore showing that

$$T\sum_{j=1}^{m} \hat{\lambda}^*_{ij} = T\sum_{j=1}^{m} \hat{\lambda}_{ij} + o_p(1) \text{ as } T, N \to \infty,$$

will be sufficient for $\mathrm{LR}^{\mathrm{SL}^*}_{\mathrm{trace_{iT}}}(0)$ to be asymptotically equivalent to $\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0)$. In order to prove the above relation we need the following results for the moment matrices $M_{i,jk}$ and $M^*_{i,jk}$, $j,k = 0,1,2$.

Lemma A.3. *For the moment matrices* $M_{i,jk}$ *and* $M^*_{i,jk}$, $j,k = 0,1,2$; $i = 1,\ldots,N$ *it holds that:*

(i) $\left\|M^*_{i,00} - M_{i,00}\right\| = m\, O_p\left(C^{-1}_{NT}\right)$,

(ii) $\left\|M^*_{i,01} - M_{i,01}\right\| = m\, O_p\left(C^{-1}_{NT}\right)$,

(iii) $\left\|M^*_{i,02} - M_{i,02}\right\| = m\sqrt{p-1}\, O_p\left(C^{-1}_{NT}\right)$,

(iv) $\left\|M^*_{i,12} - M_{i,12}\right\| = mp^{3/2}\, O_p\left(C^{-1}_{NT}\right)$,

(v) $\left\|M^*_{i,22} - M_{i,22}\right\| = m(p-1)\, O_p\left(C^{-1}_{NT}\right)$,

(vi) $\left\|\left(M^*_{i,22}\right)^{-1} - M^{-1}_{i,22}\right\| = m(p-1)\, O_p\left(C^{-1}_{NT}\right)$,

(vii) $\left\| \frac{1}{T} \left(M^*_{i,11} - M_{i,11} \right) \right\| = m\, O_p \left(C_{NT}^{-1} \right).$

Proof. (i) By the definition of $M^*_{i,00}$ and $M_{i,00}$ and Lemma A.2 (iii) we get

$$\begin{aligned} \left\| M^*_{i,00} - M_{i,00} \right\| &= \left\| \frac{1}{T} \sum_{t=2}^{T} \left(Z^*_{i,0t} Z^{*\prime}_{i,0t} - Z_{i,0t} Z'_{i,0t} \right) \right\| \\ &= \left\| \frac{1}{T} \sum_{t=2}^{T} \left(\Delta \tilde{X}^*_{it} \Delta \tilde{X}^{*\prime}_{it} - \Delta \tilde{X}_{it} \Delta \tilde{X}'_{it} \right) \right\| \\ &= m\, O_p \left(C_{NT}^{-1} \right). \end{aligned}$$

(ii) Similarly, by the definition of $M^*_{i,01}$ and $M_{i,01}$ and Lemma A.2 (iv) we have

$$\begin{aligned} \left\| M^*_{i,01} - M_{i,01} \right\| &= \left\| \frac{1}{T} \sum_{t=2}^{T} \left(Z^*_{i,0t} Z^{*\prime}_{i,1t} - Z_{i,0t} Z'_{i,1t} \right) \right\| \\ &= \left\| \frac{1}{T} \sum_{t=2}^{T} \left(\Delta \tilde{X}^*_{it} \tilde{X}^{*\prime}_{i,t-1} - \Delta \tilde{X}_{it} \tilde{X}'_{i,t-1} \right) \right\| \\ &= m\, O_p \left(C_{NT}^{-1} \right). \end{aligned}$$

(iii)

$$\begin{aligned} &\left\| M^*_{i,02} - M_{i,02} \right\| \\ &= \left\| \frac{1}{T} \sum_{t=2}^{T} \left(Z^*_{i,0t} Z^{*\prime}_{i,2t} - Z_{i,0t} Z'_{i,2t} \right) \right\| \\ &= \left\| \frac{1}{T} \sum_{t=p_i+1}^{T} \left((Z^*_{i,0t} - Z_{i,0t})(Z^*_{i,2t} - Z_{i,2t})' + (Z^*_{i,0t} - Z_{i,0t}) Z'_{i,2t} + Z_{i,0t} (Z^*_{i,2t} - Z_{i,2t})' \right) \right\| \\ &\leq \left\| \frac{1}{T} \sum_{t=p_i+1}^{T} (Z^*_{i,0t} - Z_{i,0t})(Z^*_{i,2t} - Z_{i,2t})' \right\| + \left\| \frac{1}{T} \sum_{t=p_i+1}^{T} (Z^*_{i,0t} - Z_{i,0t}) Z'_{i,2t} \right\| \\ &\quad + \left\| \frac{1}{T} \sum_{t=p_i+1}^{T} Z_{i,0t} (Z^*_{i,2t} - Z_{i,2t})' \right\| = a + b + c. \end{aligned}$$

The last sum is dominated by the two cross-product terms b and c. Now, considering c and using the results in Lemma A.2, we obtain

$$\begin{aligned} c^2 &= \left\| \frac{1}{T} \sum_{t=p_i+1}^{T} \Delta \tilde{X}_{it} \left((\Delta \tilde{X}^{*\prime}_{i,t-1}, \ldots, \Delta \tilde{X}^{*\prime}_{i,t-p_i+1}) - (\Delta \tilde{X}'_{i,t-1}, \ldots, \Delta \tilde{X}'_{i,t-p_i+1}) \right) \right\|^2 \\ &\leq \frac{1}{T} \sum_{t=p_i+1}^{T} \left\| \Delta \tilde{X}_{it} \right\|^2 \sum_{h=1}^{p_i-1} \frac{1}{T} \sum_{t=p_i+1}^{T} \left\| \Delta \tilde{X}^*_{i,t-h} - \Delta \tilde{X}_{i,t-h} \right\|^2 \\ &\leq m^2 (p-1)\, O_p \left(C_{NT}^{-2} \right). \end{aligned}$$

The same result can be shown to hold for b. Therefore $\left\|M^*_{i,02} - M_{i,02}\right\| = m\sqrt{p-1}\,O_p\left(C_{NT}^{-1}\right).$

(iv) Following the proof of Lemma F.2 from the Appendix in Bai and Ng (2004), we have

$$
\begin{aligned}
&\left\|M^*_{i,12} - M_{i,12}\right\|^2 \\
&= \left\|\frac{1}{T}\sum_{t=p_i+1}^{T}\left(Z^*_{i,1t}Z^{*\prime}_{i,2t} - Z_{i,1t}Z'_{i,2t}\right)\right\|^2 \\
&= \left\|\frac{1}{T}\sum_{t=p_i+1}^{T}\left(\tilde{X}^*_{i,t-1}\left(\Delta\tilde{X}^{*\prime}_{i,t-1},\ldots,\Delta\tilde{X}^{*\prime}_{i,t-p_i+1}\right) - \tilde{X}_{i,t-1}\left(\Delta\tilde{X}'_{i,t-1},\ldots,\Delta\tilde{X}'_{i,t-p_i+1}\right)\right)\right\|^2 \\
&= \sum_{j=1}^{p_i-1}\left\|\frac{1}{T}\sum_{t=p_i+1}^{T}\left(\tilde{X}^*_{i,t-1}\Delta\tilde{X}^{*\prime}_{i,t-j} - \tilde{X}_{i,t-1}\Delta\tilde{X}'_{i,t-j}\right)\right\|^2.
\end{aligned}
$$

Since for each j, $1 \le j \le p_i$, $\tilde{X}_{i,t-1}$ can be represented as

$$
\tilde{X}_{i,t-1} = \tilde{X}_{i,t-j-1} + \Delta\tilde{X}_{i,t-j} + \ldots + \Delta\tilde{X}_{i,t-1},
$$

from Lemma A.2 (iii) and (iv) and by similar arguments as in the proof of Lemma B.1 *(iii)* of Bai and Ng (2004) it follows that for the $j-$th summand above we have

$$
\begin{aligned}
&\frac{1}{T}\sum_{t=p_i+1}^{T}\left(\tilde{X}^*_{i,t-1}\Delta\tilde{X}^{*\prime}_{i,t-j} - \tilde{X}_{i,t-1}\Delta\tilde{X}'_{i,t-j}\right) \\
&= \frac{1}{T}\sum_{t=p_i+1}^{T}\left(\tilde{X}^*_{i,t-j-1}\Delta\tilde{X}^{*\prime}_{i,t-j} - \tilde{X}_{i,t-j-1}\Delta\tilde{X}'_{i,t-j}\right) \\
&\quad + \sum_{h=1}^{j}\frac{1}{T}\sum_{t=p_i+1}^{T}\left(\Delta\tilde{X}^*_{i,t-h}\Delta\tilde{X}^{*\prime}_{i,t-j} - \Delta\tilde{X}_{i,t-h}\Delta\tilde{X}'_{i,t-j}\right) \\
&\le m\,O_p\left(C_{NT}^{-1}\right) + jm\,O_p\left(C_{NT}^{-1}\right) = m(j+1)\,O_p\left(C_{NT}^{-1}\right).
\end{aligned}
$$

We thus obtain

$$
\begin{aligned}
\left\|M^*_{i,12} - M_{i,12}\right\|^2 &\le \sum_{j=1}^{p_i-1}(j+1)^2 m^2\,O_p\left(C_{NT}^{-2}\right) \le p^3 m^2\,O_p\left(C_{NT}^{-2}\right), \\
\left\|M^*_{i,12} - M_{i,12}\right\| &\le m p^{3/2}\,O_p\left(C_{NT}^{-1}\right).
\end{aligned}
$$

(v) The proof follows that of (iii) and is thus omitted.

(vi) First note that by Lemma A3 *(iii)* of Saikkonen and Lütkepohl (2000) $M_{i,22}$ converges in probability to the autocovariance matrix of the stationary series $Z_{i,2t}$, which we shall denote by $M^A_{i,22}$. Thus $\left\|M^A_{i,22}\right\| = O_p(1)$, $\left\|(M^A_{i,22})^{-1}\right\| = O_p(1)$ and $\left\|M_{i,22} - M^A_{i,22}\right\| = o_p(1)$ assuming that p and m remain fixed as $T \to \infty$. Now, using the same arguments as in Lemma C.1 *(ii)* of the Appendix in Bai and Ng (2004), we have that

$$\begin{aligned}\left\|\left(M_{i,22}^{*}\right)^{-1}-M_{i,22}^{-1}\right\| &= \left\|\left(M_{i,22}^{*}\right)^{-1}\left(M_{i,22}-M_{i,22}^{*}\right)M_{i,22}^{-1}\right\| \\ &\leq \left(\left\|\left(M_{i,22}^{*}\right)^{-1}-M_{i,22}^{-1}\right\|+\left\|M_{i,22}^{-1}\right\|\right)\left\|M_{i,22}-M_{i,22}^{*}\right\|\left\|M_{i,22}^{-1}\right\|,\end{aligned}$$

which after re-arranging terms becomes

$$\left\|\left(M_{i,22}^{*}\right)^{-1}-M_{i,22}^{-1}\right\| \leq \frac{\left\|M_{i,22}-M_{i,22}^{*}\right\|\left\|M_{i,22}^{-1}\right\|^{2}}{1-\left\|M_{i,22}-M_{i,22}^{*}\right\|\left\|M_{i,22}^{-1}\right\|}. \tag{A.16}$$

For $\left\|M_{i,22}^{-1}\right\|$ we have that

$$\left\|M_{i,22}^{-1}\right\| \leq \left\|M_{i,22}^{-1}-(M_{i,22}^{A})^{-1}\right\|+\left\|(M_{i,22}^{A})^{-1}\right\|. \tag{A.17}$$

Following the proof of Theorem 4.1 of Said and Dickey (1984), denoting $\left\|(M_{i,22}^{A})^{-1}\right\| = p$ and $\left\|M_{i,22}^{-1}-\left(M_{i,22}^{A}\right)^{-1}\right\| = q$, for the first term on the RHS of (A.17) we obtain

$$\begin{aligned}q = \left\|M_{i,22}^{-1}-\left(M_{i,22}^{A}\right)^{-1}\right\| &= \left\|M_{i,22}^{-1}\left(M_{i,22}^{A}-M_{i,22}\right)\left(M_{i,22}^{A}\right)^{-1}\right\| \\ &\leq \left\|M_{i,22}^{-1}\right\|\left\|M_{i,22}^{A}-M_{i,22}\right\|\left\|\left(M_{i,22}^{A}\right)^{-1}\right\| \\ &\leq (p+q)\left\|M_{i,22}^{A}-M_{i,22}\right\|p. \\ \Leftrightarrow q &\leq \frac{p^{2}\left\|M_{i,22}^{A}-M_{i,22}\right\|}{1-p\left\|M_{i,22}^{A}-M_{i,22}\right\|} = \frac{o_p(1)}{O_p(1)} = o_p(1).\end{aligned}$$

Thus

$$\left\|M_{i,22}^{-1}\right\| \leq o_p(1)+O_p(1) = O_p(1),$$

and substituting this result into (A.16) we obtain that

$$\left\|\left(M_{i,22}^{*}\right)^{-1}-M_{i,22}^{-1}\right\| \leq O_p(1)\left\|M_{i,22}-M_{i,22}^{*}\right\| = m(p-1)\,O_p\left(C_{NT}^{-1}\right).$$

The proof of (vii) follows directly from the definition of $M_{i,11}$ and $M_{i,11}^{*}$ and Lemma A.2 (ii). □

Lemma A.4. *For each $i = 1, \ldots, N$ it holds that*

(i) $\left\| S^*_{i,01} - S_{i,01} \right\| = p^{3/2} m \, O_p\left(C_{NT}^{-1}\right),$

(ii) $\left\| S^*_{i,10} - S_{i,10} \right\| = p^{3/2} m \, O_p\left(C_{NT}^{-1}\right),$

(iii) $\left\| S^*_{i,00} - S_{i,00} \right\| = m(p-1) \, O_p\left(C_{NT}^{-1}\right),$

(iv) $\left\| \left(S^*_{i,00}\right)^{-1} - S^{-1}_{i,00} \right\| = m(p-1) \, O_p\left(C_{NT}^{-1}\right),$

(v) $\left\| \frac{1}{T} S^*_{i,11} - \frac{1}{T} S_{i,11} \right\| = p^{3/2} m \, O_p\left(C_{NT}^{-1}\right).$

Proof. The results in (i), (ii), (iii) and (v) follow from the definition of the matrices $S_{i,jk}, S^*_{i,jk}$, $j, k = 0, 1$ and Lemma A.3. The proof of (iv) uses the same argument as that of Lemma A.3 (vi), given that $S_{i,00}$ converges in probability to the positive definite matrix $\Sigma_{00} := Var\left(\Delta X_{it} | \Delta X_{i,t-1}, \ldots, \Delta X_{i,t-p+1}\right)$ for each $i = 1, \ldots, N$ (Johansen (1995), Lemma 10.1 and Saikkonen and Lütkepohl (2000), Lemma A.3). □

Proof of Theorem 3.1. Saikkonen and Lütkepohl (2000) showed that under $H_0 : r_i \leq r$ for each $i = 1, \ldots, N$ the smallest $m - r$ eigenvalues of the eigenvalue problem

$$\left| \lambda S_{i,11} - S_{i,10} S^{-1}_{i,00} S_{i,01} \right| = 0, \tag{A.18}$$

when normalized by T, converge in distribution to those of

$$\left| \lambda \int_0^1 W_*(s) W_*(s)' ds - \left(\int_0^1 W_*(s) dW_*(s)' \right) \left(\int_0^1 W_*(s) dW_*(s)' \right)' \right| = 0.$$

In the above expression $W_*(s) = W(s) - sW(1)$ denotes a $d-$dimensional Brownian bridge with $d = m - r$.

To see this, consider the ordered eigenvalues of the eigenvalue problem

$$\left| \begin{bmatrix} \frac{1}{T} \beta_i' S_{i,11} \beta_i & \frac{1}{T} \beta_i' S_{i,11} \beta_{i,\perp} \\ \frac{1}{T} \beta_{i,\perp}' S_{i,11} \beta_i & \frac{1}{T} \beta_{i,\perp}' S_{i,11} \beta_{i,\perp} \end{bmatrix} - \mu \begin{bmatrix} \beta_i' S_{i,10} S^{-1}_{i,00} S_{i,01} \beta_i & \beta_i' S_{i,10} S^{-1}_{i,00} S_{i,01} \beta_{i,\perp} \\ \beta_{i,\perp}' S_{i,10} S^{-1}_{i,00} S_{i,01} \beta_i & \beta_{i,\perp}' S_{i,10} S^{-1}_{i,00} S_{i,01} \beta_{i,\perp} \end{bmatrix} \right| = 0, \tag{A.19}$$

which are

$$\hat{\mu}_{i,1} = \frac{1}{T \hat{\lambda}_{i,m}}, \ \ldots, \ \hat{\mu}_{i,m} = \frac{1}{T \hat{\lambda}_{i,1}}.$$

By the same arguments as in Lemma 6 of Johansen (1988), and using the limiting results of Saikkonen and Lütkepohl (2000) regarding the GLS-detrended cross-sectionally independent processes $\tilde{X}_{it}$, it can be shown that the $m-r$ largest μ_i's converge in distribution to the ordered eigenvalues of

$$\left| \int_0^1 W_*(s)W_*(s)'ds - \mu \left(\int_0^1 W_*(s)dW_*(s)' \right) \left(\int_0^1 W_*(s)dW_*(s)' \right)' \right| = 0. \tag{A.20}$$

Under the null hypothesis of no cointegration, $\beta_i = 0$, $\beta_{i,\perp} = I_m$, and (A.19) simplifies to

$$\left| \frac{1}{T} S_{i,11} - \mu S_{i,10} S_{i,00}^{-1} S_{i,01} \right| = 0, \tag{A.21}$$

and all its solutions converge in distribution to those of (A.20), where $W_*(s)$ is an m-dimensional Brownian bridge.

However, the same result holds also for all eigenvalues $\hat{\lambda}_{i,1}^*, \ldots, \hat{\lambda}_{i,m}^*$ of the eigenvalue problem based on the defactored data,

$$\left| \lambda S_{i,11}^* - S_{i,10}^* \left(S_{i,00}^* \right)^{-1} S_{i,01}^* \right| = 0, \tag{A.22}$$

because by Lemma A.4 each element of

$$\left| \frac{1}{T} S_{i,11}^* - \mu S_{i,10}^* \left(S_{i,00}^* \right)^{-1} S_{i,01}^* \right| = 0, \tag{A.23}$$

converges to the corresponding element of (A.21) at rate $O_p\left(C_{NT}^{-1}\right)$, assuming that m and p remain fixed as $T, N \to \infty$. Since the ordered eigenvalues are continuous functions of the coefficients, it holds that the solutions of (A.23)

$$\hat{\mu}_{i,1}^* = \frac{1}{T\hat{\lambda}_{i,m}^*}, \ldots, \hat{\mu}_{i,m}^* = \frac{1}{T\hat{\lambda}_{i,1}^*},$$

converge to those of (A.21) by the Continuous Mapping Theorem, i.e.

$$\hat{\mu}_{i,j}^* = \hat{\mu}_{i,j} + O_p\left(C_{NT}^{-1}\right), \quad j = 1, \ldots, m, \tag{A.24}$$

and are thus also bounded in probability. The rate of convergence $O_p(C_{NT}^{-1})$ in (A.24) is preserved by the Lipschitz Mapping Theorem (see Whitt (2002, pp. 85)) because the mapping $A \mapsto \lambda_i(A)$, $i = 1, \ldots, n$, is Lipschitz continuous on the space of Hermitian matrices (Tao, 2012, pp. 47). As shown by Johansen (1995, pp. 95), the solutions to the eigenvalue problem (A.18) are the same as the eigenvalues of the matrix $S_{i,11}^{-1/2} S_{i,10} S_{i,00}^{-1} S_{i,01} S_{i,11}^{-1/2}$ which is symmetric and with real entries.

Now, rewriting (A.24) in terms of $\hat{\lambda}_{i,1}^*, \ldots, \hat{\lambda}_{i,m}^*$ and noting that under the null of no cointegration $|\hat{\lambda}_{i,j}| = |\hat{\lambda}_{i,j}^*| = O_p\left(T^{-1}\right)$ by Proposition A.1 for $j = 1, \ldots, m$, we obtain that

$$T\left(\hat{\lambda}_{i,j} - \hat{\lambda}_{i,j}^*\right) = O_p\left(C_{NT}^{-1}\right),$$

which yields that $\mathrm{LR}_{\mathrm{trace}_{iT}}^{\mathrm{SL}*}(0) = \mathrm{LR}_{\mathrm{trace}_{iT}}^{\mathrm{SL}}(0) + O_p\left(C_{NT}^{-1}\right)$.

In order to prove the consistency of the test based on $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace}_{iT}}(0)$, we need to establish that it diverges to $+\infty$ under the alternative. Let $r_i > 0$ be a point in the alternative, i.e. there exists a non-zero $(m \times r_i)$ matrix β_i of full rank such that $\beta_i' \tilde{X}_{it}$ is $I(0)$. We argue as in the first part of the proof of Theorem 11.1 of Johansen (1995) and write the eigenvalue problem (A.22) in the directions β_i and $\frac{1}{\sqrt{T}}\beta_{i,\perp}$ as in (A.19). In the limit as $T \to \infty$ by Lemma A.4 we obtain

$$\left|\lambda \begin{bmatrix} \beta_i' S^*_{i,11}\beta_i & \frac{1}{\sqrt{T}}\beta_i' S^*_{i,11}\beta_{i,\perp} \\ \frac{1}{\sqrt{T}}\beta_{i,\perp}' S^*_{i,11}\beta_i & \frac{1}{T}\beta_{i,\perp}' S^*_{i,11}\beta_{i,\perp} \end{bmatrix} - \begin{bmatrix} \Sigma_{i,\beta 0}\Sigma_{i,00}^{-1}\Sigma_{i,0\beta} & 0 \\ 0 & 0 \end{bmatrix}\right| = 0, \tag{A.25}$$

where $\Sigma_{i,\beta 0}$, $\Sigma_{i,0\beta}$ and $\Sigma_{i,00}$ are constant positive definite matrices defined as the probability limits of the covariance matrices of the stationary and ergodic processes $\Delta\tilde{X}_{it}$ and $\beta'\tilde{X}_{i,t-1}$ as in Johansen (1995, pp. 141).

(A.25) can be written as

$$\left|\lambda\beta_i' S^*_{i,11}\beta_i - \Sigma_{i,\beta 0}\Sigma_{i,00}^{-1}\Sigma_{i,0\beta}\right| \times$$
$$\left|\lambda\frac{1}{T}\beta_{i,\perp}' S^*_{i,11}\beta_{i,\perp} - \lambda^2\left[\frac{1}{\sqrt{T}}\beta_i' S^*_{i,11}\beta_{i,\perp}\left(\lambda\beta_i' S^*_{i,11}\beta_i - \Sigma_{i,\beta 0}\Sigma_{i,00}^{-1}\Sigma_{i,0\beta}\right)^{-1}\frac{1}{\sqrt{T}}\beta_{i,\perp}' S^*_{i,11}\beta_i\right]\right| = 0.$$

The largest solutions are those of

$$\left|\lambda\beta_i' S^*_{i,11}\beta_i - \Sigma_{i,\beta 0}\Sigma_{i,00}^{-1}\Sigma_{i,0\beta}\right| = 0, \tag{A.26}$$

for which it holds that

$$\|\beta_i' S^*_{i,11}\beta_i\| = O_p(1) + O_p\left(\frac{\sqrt{T}}{\sqrt{N}}\right) + O_p\left(\frac{T}{N}\right), \tag{A.27}$$

and also that

$$\|(\beta_i' S^*_{i,11}\beta_i)^{-1}\| \geq O_p\left(\frac{1}{1 + \frac{\sqrt{T}}{\sqrt{N}} + \frac{T}{N}}\right). \tag{A.28}$$

This implies that the largest solutions $\hat{\lambda}^*_1, \ldots, \hat{\lambda}^*_{r_i}$ are $O_p(1)$ if $T/N \to 0$ or $T/N \to c$ for some constant $c > 0$, and $O_p(N/T)$ if $T/N \to \infty$ as $T, N \to \infty$ simultaneously. Therefore $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace_{iT}}}(0)$ diverges to $+\infty$ at rate $O_p\left((\min(N,T)\right)$.

□

Proof of Theorem 3.2. Note that the $O_p\left(C_{NT}^{-1}\right)$ term in the differences between $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace}_{iT}}(0)$ and $\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace}_{iT}}(0)$, $i = 1, \ldots, N$, arises from the $O_p\left(C_{NT}^{-1}\right)$ terms in the difference between the cross-products of $\tilde{X}^*_{it}(j)$ and $\tilde{X}^*_{it}(l)$ and their first differences in Lemma A.1. Therefore, the theorem follows if each $O_p\left(C_{NT}^{-1}\right)$ term from Lemma A.1 becomes $O_p\left(\sqrt{N}/T\right) + O_p\left(C_{NT}^{-1}\right)$ after averaging over the cross-sections and normalising by $\sqrt{N}$. The latter has been shown in Lemmas 1, 2, 3 and 4 of Bai and Ng (2010). □

Proof of Theorem 3.3. The proof follows the arguments of the proof of Theorem 1 of Larsson et al. (2001, pp. 136-141). Note, however, that for the standard normal limiting distribution of $\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}}}$ to hold as $T, N \to \infty$ simultaneously, a relative expansion rate of $N/T \to 0$ is required, and not $\sqrt{N}/T \to 0$.[2] Our considerations are briefly outlined below. We further note that assuming homogeneous cointegrating vectors and loading matrices across the cross-sections as in Assumption 3$'$ of Larsson et al. (2001) is not necessary, as it automatically holds under the null of no cointegration.

Larsson et al. (2001) show that the LR trace statistic for a VAR(p) process can be approximated by the LR trace statistic of the process with a single lag, whereby making errors of the order $O_p\left(T^{-1/2}\right)$. Using the results in Lemmas A.3 - A.6 of Saikkonen and Lütkepohl (1997), it can easily be shown that this order of the approximation error holds also for the GLS-detrended process $\tilde{X}_{it}$ with a VECM representation:

$$\Delta\tilde{X}_{it} = \Pi_i\tilde{X}_{it} + \sum_{j=1}^{p_i-1}\Gamma_{ij}\Delta\tilde{X}_{it} + e_{it}, \tag{A.29}$$

$$e_{it} = \varepsilon_{it} + \alpha_i\beta_i'(\tilde{\mu}_{i0} - \mu_{i0}) + \alpha_i\beta_i'(\tilde{\mu}_{i1} - \mu_{i1})(t-1) - \Gamma_i(\tilde{\mu}_{i1} - \mu_{i1}). \tag{A.30}$$

We make use of the notation (A.14) and rewrite the model as

$$Z_{i,0t} = \alpha_i\beta_i' Z_{i,1t} + \Psi_i Z_{i,2t} + e_{it},$$

where $\Psi_i \equiv (\Gamma_{i1}, \ldots, \Gamma_{i,p_i-1})$. As in Johansen (1995) we define $S_{i,1e} \equiv S_{i,10} - S_{i,11}\beta_i\alpha_i'$ and expand the expression for $S_{i,1e}$ as

$$\begin{aligned}
S_{i,1e} &= M_{i,1e} - M_{i,12}M_{i,22}^{-1}M_{i,2e}\\
&= \frac{1}{T}\sum_t Z_{i,1t}e_{i,t}' - \frac{1}{T}\sum_t M_{i,12}M_{i,22}^{-1}Z_{i,2t}e_{it}'\\
&= \frac{1}{T}\sum_t \left(X_{i,t-1} - (\tilde{\mu}_{i0} - \mu_{i0}) - (\tilde{\mu}_{i1} - \mu_{i1})(t-1)\right)e_{it}' - M_{i,12}M_{i,22}^{-1}\frac{1}{T}\sum_t Z_{i,2t}e_{it}'.
\end{aligned}$$

Note that under the null hypothesis of no cointegration $\beta_i = \alpha_i = 0$ and $\beta_{i,\perp} = \alpha_{i,\perp} = I_m$, so that $S_{i,10} \equiv S_{i,1e}$ and $e_{it} = \varepsilon_{it} - \Gamma_i(\tilde{\mu}_{i1} - \mu_{i1})$. We thus obtain

$$\begin{aligned}
S_{i,1e} = &\frac{1}{T}\sum_t X_{i,t-1}\varepsilon_{it}' - \frac{1}{T}\sum_t X_{i,t-1}\left(\Gamma_i(\tilde{\mu}_{i1} - \mu_{i1})\right)'\\
&- (\tilde{\mu}_{i1} - \mu_{i1})\frac{1}{T}\sum_t (t-1)\varepsilon_{it}' + (\tilde{\mu}_{i1} - \mu_{i1})\frac{1}{T}\sum_t (t-1)\left(\Gamma_i(\tilde{\mu}_{i1} - \mu_{i1})\right)'\\
&- (\tilde{\mu}_{i0} - \mu_{i0})\frac{1}{T}\sum_t \varepsilon_{it}' + \frac{1}{T}\sum_t (\tilde{\mu}_{i0} - \mu_{i0})\left(\Gamma_i(\tilde{\mu}_{i1} - \mu_{i1})\right)'\\
&- M_{i,12}M_{i,22}^{-1}\frac{1}{T}\sum_t Z_{i,2t}\varepsilon_{it}' + M_{i,12}M_{i,22}^{-1}\frac{1}{T}\sum_t Z_{i,2t}\left(\Gamma_i(\tilde{\mu}_{i1} - \mu_{i1})\right)'.
\end{aligned}$$

By Lemma A.6 of Saikkonen and Lütkepohl (1997) the first four terms on the RHS in the expression above converge in distribution to $C_i\Omega_i^{1/2}\int_0^1 W_{i*}(s)dW_{i*}(s)'\Omega_i^{1/2}$ for each cross-section, where W_{i*} are standard Brownian bridges which are independent across i, while the

[2] The relative expansion rate $\sqrt{N}/T \to 0$ is required in Theorem 1 of Larsson et al. (2001).

remaining terms are of the order $O_p(T^{-1/2})$, also independent across i. Denoting asymptotic equivalence by $\sim$, we therefore obtain

$$S_{i,1e} \sim C_i \Omega_i^{1/2} \int_0^1 W_{i*}(s) dW_{i*}(s)' \Omega_i^{1/2} + \frac{1}{\sqrt{T}} R_{1it}, \text{ where}$$

$$\begin{aligned}
\frac{1}{\sqrt{T}} R_{1it} &= \frac{1}{\sqrt{T}} \Bigg[-(\tilde{\mu}_{i0} - \mu_{i0}) \frac{1}{\sqrt{T}} \sum_t \varepsilon_{it} + \frac{1}{\sqrt{T}} \sum_t (\tilde{\mu}_{i0} - \mu_{i0}) \left(\Gamma_i (\tilde{\mu}_{i1} - \mu_{i1}) \right)' \\
&\quad - M_{i,12} M_{i,22}^{-1} \frac{1}{\sqrt{T}} \sum_t Z_{i,2t} \varepsilon_{it}' + M_{i,12} M_{i,22}^{-1} \frac{1}{\sqrt{T}} \sum_t Z_{i,2t} \left(\Gamma_i (\tilde{\mu}_{i1} - \mu_{i1}) \right)' \Bigg] \\
&= \frac{1}{\sqrt{T}} \left(C_i X_{i1T} + Y_{i1T} \right).
\end{aligned}$$

In the above expression, the matrices C_i, defined as $C_i = \beta_\perp \left(\alpha_\perp' \Gamma_i \beta_\perp \right)^{-1} \alpha_\perp'$, reduce to $C_i = \Gamma_i^{-1}$ under the null hypothesis, and X_{i1T} and Y_{i1T} are sequences of $O_p(1)$ random variables, which are independent across i.

Note that $\frac{1}{\sqrt{T}} R_{1it}$ are the only approximation error terms of the order $O_p(T^{-1/2})$ that arise. It is straightforward to check that $T^{-1} S_{i,11} = T^{-1} M_{i,11} - T^{-1} M_{i,12} M_{i,22}^{-1} M_{21}$ gives rise only to $O_p(T^{-1})$ terms in the limit when converging weakly to $C_i \Omega_i^{1/2} \int_0^1 W_{i*}(s) W_{i*}(s)' ds \Omega_i^{1/2} C_i'$. $S_{i,00}$ converges in probability to $\Sigma_{i,00}$, which, on the other hand, equals Ω_i under the null of no cointegration. These results follow from Lemma A.6 *(i)* and Lemma A.3 *(iii)* of Saikkonen and Lütkepohl (1997), respectively.

Therefore, under the null hypothesis of no cointegration, the $\text{LR}^{\text{SL}}_{\text{trace}_{iT}}(0)$ statistic can be written as

$$\begin{aligned}
\text{LR}^{\text{SL}}_{\text{trace}_{iT}}(0) &= T \,\text{tr} \left(S_{i,11}^{-1} S_{i,10} S_{i,00}^{-1} S_{i,01} \right) && \text{(A.31)} \\
&\sim Z_{0iT} + \frac{1}{\sqrt{T}} Z_{1iT} + O_p \left(\frac{1}{T} \right), && \text{(A.32)}
\end{aligned}$$

where Z_{0iT} and Z_{1iT} are $O_p(1)$ and independent over i, and

$$Z_{0iT} = T \,\text{tr} \left(\left(S_{i,11}^{(1)} \right)^{-1} S_{i,10}^{(1)} \left(S_{i,00}^{(1)} \right)^{-1} S_{i,01}^{(1)} \right) \Rightarrow Z_d$$

for Z_d defined as in (12). The $S_{i,jk}^{(1)}$ matrices, $j,k = 0,1$, are calculated from the VAR(1) model

$$\Delta \tilde{X}_{it} = \Pi_i \tilde{X}_{it} + e_{it},$$

where e_{it} reduces to $e_{it} = \varepsilon_{it} - (\tilde{\mu}_{i1} - \mu_{i1})$ under the null hypothesis.

The Z_{1iT} terms have the representation

$$Z_{1it} = \operatorname{tr}\left(C_i X_{iT} + Y_{iT}\right),$$
$$X_{iT} = \left(\int_0^1 W_{i*}(s)dW_{i*}(s)'\right)' \left(\int_0^1 W_{i*}(s)W_{i*}(s)'ds\right)^{-1} X_{1iT}\Sigma_{i,00}^{-1},$$
$$Y_{iT} = \left(\int_0^1 W_{i*}(s)dW_{i*}(s)'\right)' \left(\int_0^1 W_{i*}(s)W_{i*}(s)'ds\right)^{-1} Y_{1iT}\Sigma_{i,00}^{-1}.$$

Denoting $\overline{Z_{0T}} = \frac{1}{N}\sum_{i=1}^{N} Z_{0iT}$ and $\overline{Z_{1T}} = \frac{1}{N}\sum_{i=1}^{N} Z_{1iT}$, for the standardized cross-sectional average of the individual LR trace statistics we then obtain

$$\Upsilon_{\overline{\mathrm{LR}}^{\mathrm{SL}}_{\mathrm{trace}}} = \frac{\sqrt{N}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0) - \mathbb{E}\left(\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0)\right)\right)\right]}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\operatorname{Var}\left(\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace_{iT}}}(0)\right)}} \tag{A.33}$$

$$\sim \frac{\sqrt{N}\left[\frac{1}{N}\sum_{i=1}^{N}\left(Z_{0it} + \frac{1}{\sqrt{T}}Z_{1iT} + O_p\left(\frac{1}{T}\right)\right) - \mathbb{E}\left(Z_d\right)\right]}{\sqrt{\operatorname{Var}\left(Z_d\right)}} \tag{A.34}$$

$$= \frac{\sqrt{N}\left(\overline{Z_{0T}} - \mu_T\right)}{\sigma_T} + \frac{\sqrt{N}}{\sqrt{T}}\frac{\overline{Z_{1T}}}{\sigma_T} + O_p\left(\frac{\sqrt{N}}{T}\right), \tag{A.35}$$

where $\mu_T = \mathbb{E}\left(Z_{0iT}\right)$ and $\sigma_T^2 = \operatorname{Var}\left(Z_{0iT}\right)$, $\forall i$, and the order of the approximation error by standardizing with the moments of the asymptotic trace statistic Z_d is at most $O_p(T^{-1})$ (see Larsson et al., 2001, Lemma 2) and it is thus included in the $O_p(\sqrt{N}/T)$ term above. The existence and finiteness of the moments μ_T and σ_T^2 has been established in Theorem 1 of Örsal and Droge (2012). Using the arguments of Larsson et al. (2001, pp. 140-141) it can then be shown that the first term of (A.35) gives the required $N(0,1)$ limiting distribution, while $\overline{Z_{1T}}$ remains bounded in probability. Therefore, provided that $N/T \to 0$ as $T, N \to \infty$ simultaneously, the last two terms in (A.35) become zero asymptotically, which completes the proof. □

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