Socioeconomic status and word problem solving in PISA: The role of mathematical content areas

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Published in:
Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education

Publication date:
2022

Document Version
Publisher's PDF, also known as Version of record

Proceedings

of the 45th conference of the international group for the psychology of mathematics education

July 18-23, 2022

EDITORS

Ceneida Fernández / Salvador Llinares
Ángel Gutiérrez / Núria Planas

VOLUME 4

Research Reports (Si – Z)
Oral Communications, Poster Presentations

PME 45
Alicante · Spain · 2022

Universitat d'Alacant
Universidad de Alicante

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(PME) Conference
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Website:
https://web.ua.es/pme45

Cite as:

Fernández, C., Llinares, S., Gutiérrez, A., & Planas, N. (Eds.) (2022). *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Alicante, Spain: PME.

Website: https://web.ua.es/pme45

Proceedings are also available on the IGPME website:
http://www.igpme.org

ISBN: 978-84-1302-178-2

Legal Deposit: A 291-2022

ISSN 0771-100X

Printed by Universidad de Alicante.

Logo designed by the Gabinete de Imagen y Comunicación Gráfica (Universidad de Alicante)

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RESEARCH REPORTS (Si – Z)

SOCIOECONOMIC STATUS AND WORD PROBLEM SOLVING IN PISA: THE ROLE OF MATHEMATICAL CONTENT AREAS

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Mathematics performance and socioeconomic status (SES) are positively related, but the reasons are not well understood. Moreover, the strength of the relationship in large-scale assessments like PISA differs between countries, for example, between Finland and Germany. In the PISA studies, mathematical word problems are used, which cover four mathematical content areas. In the present study, we reanalyzed data from PISA 2003 to 2018 to investigate whether word problems in these content areas were related differently to SES across the two countries. The results suggest that the relationship can be attributed to different content areas in both countries. This emphasizes the importance of considering item characteristics when addressing the relationship between SES and mathematical word problem solving.

INTRODUCTION

It is a common observation that students' mathematics achievement is related to their *socioeconomic status* (SES; e.g., Martins & Veiga, 2010). For example, this is apparent in the *Programme for International Student Assessment* (PISA) studies, which show that such a relationship is found across all participating OECD countries (e.g., OECD, 2013). In 2012, when mathematics was the focus of PISA most recently, the achievement gap in mathematics between the top and bottom quarter of students with regard to SES was 90 points, which is equivalent to more than two years of schooling (OECD, 2013).

Although a positive relationship between mathematics achievement and SES is found universally, its extent differs substantially between countries. For example, in 2012, Finland and Estonia were the only participating European OECD countries that combined a high performance in mathematics and low relationship between mathematics achievement and SES (OECD, 2013). On the other hand, students in Germany and Belgium also performed above the OECD average in mathematics, but in combination with a relationship between mathematics achievement and SES that was higher than the OECD average.

Several studies have investigated possible causes for the relationship between mathematics achievement and SES (see Hopfenbeck et al., 2018, for an overview in the context of PISA). Gustafsson et al. (2018) show that SES at school level is a relevant predictor of the relationship. On the classroom level, Yang Hansen and Striethold (2018) analyzed the role of opportunities to learn but found no substantial

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 59-66). PME.

evidence that they were a relevant mediator between SES and mathematics achievement. On an individual level, Prediger et al. (2018) showed that language proficiency explained a substantial amount of the relationship in a high-stakes mathematics test.

These and other studies show that the relationship between mathematics achievement and SES is probably multicausal in nature and rooted on various levels, ranging from the educational system to individual learning processes. However, previous research has not yet focused on the level of the mathematics tasks. In large-scale assessment studies like PISA, the tasks that are used to assess mathematical achievement can be considered *complex word problems* (Strohmaier et al., 2021). These tasks typically embed a mathematical problem in a realistic context by enriching it with additional text and visual representations. Complex word problems include plenty of task characteristics that might cause an influence of SES. For example, the use of academic language features could provide additional challenges for students that are less exposed to this register in their everyday life (Prediger et al., 2018). The context in which a mathematical task is embedded might also provide students with specific advantages and disadvantages with regard to their social background (e.g., Carraher et al., 1985), but also the mathematical content of a task might be a relevant factor. Everyday arithmetic abilities may be considered the foundation for any further mathematical abilities and therefore teachers, students, and parents might regard them as the highest priority for students from all backgrounds. According to the PISA framework, the content area *Quantity* "may be the most pervasive and essential mathematical aspect of engaging with, and functioning in, our world" (OECD 2019, p. 85). On the other hand, more academic mathematical areas like formal proofs, statistics, or functional thinking might be regarded to be more relevant in academic contexts and to serve a propaedeutic role in preparing for higher secondary and tertiary education, which might be of higher priority in high-SES families and schools. Importantly, these differences between content areas might differ between countries and educational systems, depending on school systems, prevailing beliefs, and values. For example, they might be reinforced by a multi-track school system like in Germany, where curricula might emphasize different contents in different school tracks (Skopek & Passaretta, 2021). To our knowledge, no previous research has yet investigated whether performance in mathematical content areas is differentially related to SES.

The present study

In the present study, we investigated whether the content areas of the mathematical word problems that were used in the PISA studies between 2003 to 2018 can offer a more differentiated view on the relationship between SES and mathematics performance and the differences between countries. To this end, we chose Finland and Germany as examples for European countries with a relationship between mathematics achievement and SES that was lower and higher than the OECD average, respectively.

In PISA, mathematics tasks are categorized into four *content areas* (also referred to as *content categories*, *content ideas* or *overarching ideas* in the PISA assessment cycles, OECD, 2019): *Quantity*, *Space and Shape*, *Change and Relationships*, and *Uncertainty and Data*. While previous research has looked for explanations for the role of SES in learning mathematics on the institutional, classroom, and student level, we add to these findings by taking into account the content area as a task characteristic.

In line with previous analyses of the PISA datasets (e.g., OECD, 2013), we first analyzed the relationship between SES and performance in the four content areas without control variables. However, because language proficiency has shown to explain a substantial part of the relationship between SES and mathematics achievement in previous studies (Prediger et al., 2018), we included reading abilities as a measure of language proficiency as a control variable in a second step. Accordingly, we posed the following research question:

For which content areas does the relationship between SES and mathematical word problem solving in Finish and German students differ from the OECD average (with and without reading abilities as control variable)?

Overall, this approach aims at contributing to the question what causes mathematical tasks to be systematically more difficult for low-SES students, and might ultimately provide ways how to tackle the issue of social disadvantages in learning mathematics.

METHODS

Sample

This study is a secondary data analysis of the datasets from the six assessment cycles of PISA 2003 to 2018 which are publicly available via the PISA website (OECD, 2021a). No additional data were collected. Details of the sampling procedure and the number of participants are available in the technical reports by the OECD (e.g., OECD, 2021b). Across all assessment cycles between 2003 and 2018, the average number of participants was M = 5248 (SD = 609) in Germany, M = 6113 (SD = 1278) in Finland, and M = 268732 (SD = 28759) across all OECD countries. All participants were 15-year-old students.

Instruments

In the PISA studies, SES was operationalized by the *PISA index of economic, social and cultural status* (ESCS). It is a composite score based on the *highest parental occupation* (HISEI), the *parental education* (PARED) and *home possessions* (HOMEPOS), including *books in the home* (OECD, 2021b).

Throughout the six studies between 2003 and 2018, a pool of about 180 mathematical word problems have been developed to assess *mathematical literacy*. These items are embedded in a functional, real-world context and cover a variety of mathematical processes within the four content areas (OECD, 2019).

Reading abilities (*reading literacy* in PISA*)* were assessed with tasks that cover a range of different texts, processes, and scenarios. Reading literacy thus reflects a functional perspective on language abilities, situated in real-world contexts (OECD, 2019).

Data and Analyses

Even though only about 70 of the mathematics items used in PISA have been made publicly available, information about their content areas is available in the technical reports (e.g., OECD, 2021b). Because of the rotated study design in PISA, each mathematics word problem was only solved by a subset of students each year. The average number of student solutions per item per year was M = 1234 (SD = 304) in Germany, M = 2713 (SD = 689) in Finland, and M = 63044 (SD = 21585) across all OECD countries. Students' individual item solutions are available in the raw data (OECD, 2021a) and were recoded according to the technical reports as *correct* or *incorrect*.

In a first step, for each item in each assessment cycle, we conducted two logistic regressions with the item solution (incorrect/correct) as dependent variable, ESCS as independent variable (Model 1 and 2), and with reading literacy as a control variable (only Model 2). ESCS was z-standardized by the OECD sample for ease of interpretation. Consistent with the methodology used in the PISA studies, a *balanced repeated replication* (BRR) procedure was followed in order to account for the nested structure of the data (OECD, 2009). This first step was done for the Finnish subsample, the German subsample, and for the sample of all students from OECD countries. It resulted in a total of 2340 separate item analyses (390 items x 2 models x 3 subsamples).

In the second step, the regression coefficients for SES from the 2340 separate logistic regressions were then synthesized in six separate random-effects meta-analyses (Lipsey & Wilson, 2000) for Model 1 and Model 2, for each of the three subsamples. *Content area* was included as a factor (*Quantity*: 103 items, *Space and Shape*: 94 items, *Change and Relationship*: 98 items, *Uncertainty and Data*: 95 items). Coefficients were weighted based on their inverse error variance. Because most items were included in more than one assessment cycle and their coefficients might be nested, a random effect for *item* was included. Furthermore, because analyses within one assessment cycle were based on the same sample of students and therefore might be correlated, another random effect for *assessment cycle* was included. To test for statistical differences between countries, a meta-regression with dummy variables for Finland and Germany was conducted for each content area.

Finally, the reported coefficients and confidence intervals were transformed to odds ratios (*OR*s) for ease of interpretation. *OR*s indicate the multiplicative change in the solution odds (correct solution probability/incorrect solution probability) per unit change of the predictor variable. For example, an odds ratio of 1.50 means that participants who had a z-standardized ESCS of 1 (one standard deviation above OECD average) had 1.5-times or 50% higher odds of solving this task correctly than a participant with average ESCS.

We chose this two-step approach over using a comprehensive statistical model because of the complex data structure. Combining the datasets into one single logistic regression model would cause several methodological issues, including the multiple levels of nesting, missing datapoints, and weighting procedures. Meta-analysis is considered an appropriate approach to synthesize regression coefficients, provided that the regression models are sufficiently comparable (Lipsey & Wilson, 2000). The *R* packages *intsvy* (Caro & Biecek, 2017) and *metafor* (Viechtbauer, 2010) were used for analyses.

RESULTS

Results are given in Table 1. Model 1 gives the *OR*s for z-standardized ESCS on the solution probability of mathematical word problems in the four content areas. It shows that across OECD countries, students with a one standard deviation higher ESCS had 57% to 72% higher odds of correctly solving each word problem than a student with average ESCS. For Finish students, the *OR*s were significantly lower than for the OECD sample (but still significantly higher than 1) for all content areas except for *Space and Shape*, where there was no significant difference. For German students, the *OR*s were significantly higher than for the OECD sample for all content areas except *Quantity*, for which it did not differ significantly.

	Quantity		Space and Shape		Change and Relationships		Uncertainty and Data	
	OR	*CI*	*OR*	*CI*	*OR*	*CI*	*OR*	*CI*
Model 1								
OECD	1.57	1.49 - 1.65	1.60	1.52 - 1.67	1.72	1.64 - 1.80	1.59	1.50 - 1.69
Finland	**1.50**	1.15 - 1.56	1.57	1.46 - 1.69	**1.60**	1.50 - 1.71	**1.53**	1.44 - 1.61
Germany	1.60	1.51 - 1.69	**1.69**	1.58 - 1.80	**1.82**	1.70 - 1.94	**1.71**	1.61 - 1.83
Model 2								
OECD	1.14	1.10 - 1.19	1.13	1.09 - 1.18	1.19	1.15 - 1.23	1.16	1.11 - 1.21
Finland	1.15	1.11 - 1.20	**1.19**	1.14 - 1.24	1.18	1.14 - 1.23	1.18	1.13 - 1.23
Germany	**1.11**	1.09 - 1.14	**1.19**	1.14 - 1.23	1.21	1.16 - 1.27	**1.21**	1.17 - 1.26

Table 1: Odds ratios and 95% confidence intervals of z-standardized ESCS on mathematical word problem solution probability by content area. Model 2 controls for reading abilities. *OR*s printed in **bold** differ significantly ($p < .01$) from the *OR*s of the OECD sample.

In Model 2, reading abilities were included as a control variable. This substantially decreased the *OR*s for ESCS in all three samples, but still resulted in *OR*s that were significantly higher than 1, meaning that a higher ESCS was still associated with higher solution odds. Across the OECD, students with a one standard deviation higher ESCS had 13% to 19% higher odds of solving each word problem correctly compared to students with an average ESCS. For Finish students, there were no significant differences for three of the four content areas, but a higher *OR* for *Space and Shape* compared to the OECD. For German students, the *OR* was significantly lower compared to the OECD sample for *Quantity*, greater for *Space and Shape* as well as for *Uncertainty and Data*, and not significantly different for *Change and Relationships*.

DISCUSSION

Considering content areas of mathematical word problems that were used in PISA revealed a new perspective on the relationship between mathematics achievement and SES. With regard to our research question, we found that the relationship between SES and performance in the content areas differed between Finland and Germany. Without controlling for reading abilities, Finish students' mathematics performance was less influenced by SES in all content areas, although this difference was not significant for the content area *Space and Shape*. This is consistent with coarser analyses of the same data that show the overall relationship between SES and mathematics achievement is weaker in Finland compared to the OECD average (OECD, 2013). However, when controlling for reading abilities, Finland was no longer below the OECD average, and *Space and Shape* was now significantly stronger related to SES than across the OECD. Thus, it seems that while the role of SES was smaller in Finland in general, it was not explained by language proficiency to the same extent as in other OECD countries.

For Germany, the relationship between mathematics achievement and SES was higher than the OECD average before and after controlling for reading abilities for the content areas *Space and Shape* and *Uncertainty and Data*. In contrast, the relationship between SES and *Change and Relationships* was significantly different from the OECD average before, but not after controlling for reading abilities. *Quantity* even had a significantly lower relationship to SES than the OECD average when controlling for reading abilities. Across all content areas, this shows that language proficiency plays an important role in explaining why the relationship between mathematics achievement and SES is higher in Germany than across the OECD. In fact, when this role of language is taken into account, the particularly high relationship between SES and mathematics in Germany was limited to the content areas *Space and Shape* and *Uncertainty and Data*.

This seems to be an interesting starting point for investigating the processes through which the association between SES and mathematics emerges. For example, future analyses could investigate whether the different school tracks in Germany, which are associated with differences in SES (Skopek & Passaretta, 2021), offer different opportunities to learn mathematical content in the areas of *Space and Shap*e and

Uncertainty and Data, or whether the contents of these word problems offer any advantage or disadvantage for particular groups of students with regard to their SES.

The fact that the content areas of mathematical word problems were differently associated with SES in Finland and Germany raises the question where such country differences are rooted. A possible topic for future research might be the subjective importance of mathematical content areas, and whether content that is believed to be important for everyday life might be considered more relevant for all students than more theoretical, academic concepts. Similarly, including interest as a possible mediator between SES and mathematics might help to understand their relationship.

Overall, our analyses further reiterate the role that reading abilities play when investigating SES in the context of mathematics achievement (see also Prediger et al., 2018). While SES remained a positive predictor of word problem performance, controlling for reading abilities decreased the influence on SES substantially across content areas in the OECD, from 57% to 72% higher odds of a correct solution per standard deviation in SES to 13% to 19% higher odds after controlling for reading abilities.

Reanalyzing existing datasets from studies like PISA offers the benefit of a large amount of available data, but naturally, comes with limitations. The majority of items has not yet been fully published, and only their content area was available as a task characteristic. In order to investigate in detail which task characteristics play a role for the relationship with SES, studies are needed that specifically address and manipulate specific facets of mathematical word problems. At the same time, the interaction with student characteristics like interests and beliefs might provide additional insights into the role of SES in learning mathematics. Judging from the present study, a detailed look on role of the mathematical content might be an informative starting point, while taking interactions with language proficiency into account.

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