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Published in:
Econometrics and Statistics

DOI:
10.1016/j.ecosta.2016.10.001

Publication date:
2017

Document Version
Publisher's PDF, also known as Version of record

Econometrics and Statistics 2 (2017) 61–72

Contents lists available at ScienceDirect

Econometrics and Statistics

journal homepage: www.elsevier.com/locate/ecosta

Meta-analytic cointegrating rank tests for dependent panels☆

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ARTICLE INFO

Article history:
Received 5 April 2016
Revised 17 October 2016
Accepted 17 October 2016
Available online 23 November 2016

JEL Classification:
C12
C15
C33

Keywords:
Panel cointegration
p-value
Common factors
Rank test
Cross-sectional dependence

ABSTRACT

Two new panel cointegrating rank tests which are robust to cross-sectional dependence are proposed. The dependence in the data generating process is modeled using unobserved common factors. The new tests are based on a meta-analytic approach, in which the *p*-values of the individual likelihood-ratio (LR) type test statistics computed from defactored data are combined into the panel statistics. A simulation study shows that the tests have reasonable size and power properties in finite samples. The application of the tests is illustrated by investigating the monetary exchange rate model for a panel data of 19 countries.

1. Introduction

Since the beginning of the 21st century panel cointegration techniques have been widely used to test and estimate long-run macroeconomic relationships. By using time observations from different cross-sections, it is possible to increase the power of the conventional cointegration tests. However, the cross-sectional dependencies within the macro-panels should be taken into account to avoid wrong statistical inference.

There are mainly two different types of panel cointegration tests in the literature. The first type of tests are called residual-based tests and the second type of tests are called system tests. The latter ones have some advantages in comparison to the former ones. The system tests are not only suitable to find out the number of cointegrating relations, i.e. the cointegrating rank of the system, but also the test decisions are invariant to the variable used to normalize the long-run relationship.

In order to use the advantages of the system tests, our aim is to develop new panel cointegrating rank tests which allow for cross-sectional dependence.

In this study the testing procedure outlined in Arsova and Örsal (2016) is followed to propose new panel cointegration tests. Arsova and Örsal (2016) base their testing procedure on the panel analysis of nonstationarity in idiosyncratic and

☆ The data which is used for the empirical analysis is available online as supplementary material.

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http://dx.doi.org/10.1016/j.ecosta.2016.10.001

common components (PANIC) approach of Bai and Ng (2004). They propose a panel cointegrating rank test which is the standardized version of the average individual LR-type test statistics of Saikkonen and Lütkepohl (2000) computed from defactored data.

In contrast to Arsova and Örsal (2016), the testing procedure in this study is based on the approach of Maddala and Wu (1999) and Choi (2001), in which the new panel test statistics are based on combining the p-values of the individual Saikkonen and Lütkepohl LR statistics using defactored data.

In general, the panel tests based on combining p-values have several advantages in comparison to the tests based on standardizing the average of the individual test statistics. The former approach allows having a much more heterogeneous structure in the panel. Within this heterogeneous structure different deterministic terms can be included into the data generating process (DGP) of each cross-section and also the lag order can vary over cross-sections. These tests may even be applied to unbalanced panels.

Via Monte Carlo simulations we compare the finite-sample properties of our new tests with the test of Arsova and Örsal (2016) and show that the meta-analytic tests have slightly better performance in some cases.

This paper is organized as follows. Section 2 presents the DGP and the assumptions of the new panel cointegration tests. Section 3 explains the testing procedure. Section 4 presents the finite-sample properties of the proposed panel cointegration tests and compares them with others existing in the literature. Section 5 checks the validity of the monetary exchange rate model. Finally, Section 6 concludes.

Throughout the paper L and Δ represent the lag and differencing operators, respectively. $M < \infty$ denotes a generic constant which is independent of the dimensions of the panel N and T.

2. Model

The new panel cointegration tests are based on the same DGP as in Arsova and Örsal (2016):

$$Y_{it}^{cd} = Y_{it} + \Lambda_i' F_t, \quad i = 1, \dots, N, \quad t = 1, \dots T, \tag{1}$$

$$Y_{it} = \mu_{0i} + \mu_{1i} t + X_{it}, \tag{2}$$

$$X_{it} = A_{i1} X_{i,t-1} + \dots + A_{i,\bar{p}_i} X_{i,t-\bar{p}_i} + \varepsilon_{it} \quad \text{and} \tag{3}$$

$$(1 - L) F_t = C(L) u_t \quad \text{with} \quad C(L) = \sum_{j=0}^{\infty} C_j L^j, \tag{4}$$

where the m-dimensional vector $Y_{it}^{cd} = (Y_{i,1t}^{cd}, \dots, Y_{i,mt}^{cd})'$ denotes the observed cross-sectionally dependent data for unit i. Note that this model is the vector-valued extension of the model of Bai and Ng (2004). Cross-sectional dependence is allowed for through the $(k \times 1)$ vector of unobserved common factors F_t. Due to the $(k \times m)$-dimensional matrix of individual-specific factors loadings Λ_i, some factors may not influence all the cross-sections. The common factors may be either stationary, non-stationary or a combination of stationary and non-stationary processes.

In Eq. (2) μ_{0i} and μ_{1i} denote the parameters of the heterogeneous deterministic terms. X_{it} is a vector of unobserved idiosyncratic components which has a VAR representation (see Eq. (3)), whose lag order $\bar{p}_i$ may differ over cross-sections. The components of the X_{it} process can be integrated at most of order one and they are cointegrated with cointegrating rank r_i for $0 \le r_i \le m$. The error terms ε_{it} follow a martingale difference sequence, where $E(\varepsilon_{it} | \varepsilon_{is}, s < t) = 0$ and $E(\varepsilon_{it}\varepsilon_{it}' | \varepsilon_{is}, s < t) = \Omega_i$ with Ω_i being a positive definite matrix for $i = 1, \dots, N$. The ε_{it}'s are neither serially correlated nor cross-sectionally dependent. In other words, the sole source of cross-sectional dependence within the panel is the common component $\Lambda_i' F_t$. We assume that the number of common factors is known. In practice it can be determined by the information criteria of Bai and Ng (2002) or Onatski (2010).

The test is built on the VECM representation of X_{it}:

$$\Delta X_{it} = \Pi_i X_{i,t-1} + \sum_{j=1}^{\bar{p}_{i-1}} \Gamma_{ij} \Delta X_{i,t-j} + \varepsilon_{it}, \quad t = \bar{p}_i + 1, \dots, T, \quad i = 1, \dots, N, \tag{5}$$

where $\Gamma_{ij} = -(A_{i,j+1} + \dots + A_{i,\bar{p}_i})$. The $(m \times m)$ matrix $\Pi_i = -(I_m - A_{i1} - \dots, -A_{i,\bar{p}_i})$ is the cointegrating matrix for each cross-section which can be decomposed as $\Pi_i = \alpha_i \beta_i'$ with α_i and β_i being full rank $(m \times r_i)$ matrices.

Assumptions:

1. The assumptions on the common factors are:
   (a) $u_t \sim iid(0, \Sigma_u)$, $E\|u_t\|^4 \le M < \infty$.
   (b) $Var(\Delta F_t) = \sum_{j=0}^{\infty} C_j \Sigma_u C_j' > 0$.
   (c) $\sum_{j=0}^{\infty} j \|C_j\| < M < \infty$.
   (d) $C(1)$ has rank k_1, $0 \le k_1 \le k$.
2. The assumptions on the factor loadings are:

(a) Λ_i is deterministic and $\|\Lambda_i\| \leq M < \infty$, or Λ_i is stochastic and $E\|\Lambda_i\|^4 \leq M < \infty$.
(b) $N^{-1}\sum_{i=1}^{N}\Lambda_i\Lambda_i' \xrightarrow{p} \Sigma_\Lambda$ as $N \to \infty$, where Σ_Λ is a $(k \times k)$ non-random positive definite matrix.

3. Λ_i, u_t and ε_{it} are mutually independently distributed across i and t.

3. Testing procedure

Before testing for the panel cointegrating rank, the cross-sectional dependence within the panel should be eliminated. Therefore, as a first step, the data are defactored using the PANIC approach of Bai and Ng (2004). In PANIC the common components are estimated using principal components. A detailed description of the way how the factors and loadings are estimated can be found in Arsova and Örsal (2016). By subtracting the estimates of the common components, i.e. $\hat{\Lambda}_i'\hat{F}_t$, from the observed data, the cross-sectional dependence is removed from the panel. In the next step, the GLS-based LR-type cointegration test of Saikkonen and Lütkepohl (2000) is employed on the defactored data for each panel unit separately. Finally, the corresponding p-values of the individual test statistics are computed by the response surface approach outlined in Trenkler (2008).

The null and alternative hypotheses under consideration are:

$$H_0 : r_i = r = 0,\ \forall i, \quad \text{versus} \quad H_1 : r_i > 0 \quad \text{for some } i. \tag{6}$$

We propose the following panel cointegration test statistics based on a standardized version of Fisher's χ^2 p-value test and the inverse normal test, respectively:

$$P_N^* = \frac{-2\sum_{i=1}^{N}\ln(p_i^*) - 2N}{\sqrt{4N}} \quad \text{and} \tag{7}$$

$$P_{\Phi^{-1}}^* = \frac{\sum_{i=1}^{N}\Phi^{-1}(p_i^*)}{\sqrt{N}}. \tag{8}$$

Here p_i^* denotes the p-value of the Saikkonen and Lütkepohl LR-type statistic under the null hypothesis of no cointegration for individual i (henceforth $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace,iT}}(0)$) and $\Phi(.)$ denotes the cumulative distribution function of the standard normal distribution. The LR-type statistics can be computed using either the estimated idiosyncratic component $\hat{X}_{it} = \sum_{s=2}^{t}(y_{is} - \hat{\Lambda}_i'\hat{F}_t)$ for $t = 2, \ldots, T$ and $\hat{X}_{i1} = 0$, where $y_{it} = \Delta Y_{it}^{cd} - \frac{1}{T-1}\sum_{t=2}^{T}\Delta Y_{it}^{cd}$, or the defactored data $Y_{it}^* = Y_{it}^{cd} - \hat{\Lambda}_i'\hat{F}_t$. Note that the open source software JMulTi delivers the p-values of the GLS-based LR-type statistic of Saikkonen and Lütkepohl.

The limiting distribution of the proposed tests under the null and alternative hypotheses is established in the next theorem.

Theorem 1. *Under the null hypothesis of no cointegration, and when m and* $\bar{p} = \max\{\bar{p}_i | 1 \leq i \leq N\}$ *remain fixed, it holds that*

$$P_N^* \sim N(0, 1) \quad \text{and} \tag{9}$$

$$P_{\Phi^{-1}}^* \sim N(0, 1), \tag{10}$$

as $T \to \infty$ *followed by* $N \to \infty$*, or as* $T, N \to \infty$ *simultaneously with* $N/T \to 0$*. Under the alternative hypothesis the* P_N^* *statistic diverges to* $+\infty$ *and the* $P_{\Phi^{-1}}^*$ *statistic diverges to* $-\infty$*.*

Proof. This theorem is valid under the assumption that the individual statistics are computed from cross-sectionally independent data. For the proof in the sequential limits case we refer to Choi (2001), while the arguments for joint limits with $N/T \to 0$ follow that of Theorem 3.3 of Arsova and Örsal (2016) and Theorem 3 of Carrion-i-Silvestre and Surdeanu (2011). To prove the theorem for the statistics based on defactored data, the arguments of Bai and Ng (2004, p. 1176) can be followed. Let $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace,iT}}(0)$, $i = 1, \ldots, N$ be statistics based on the estimated idiosyncratic components and let $\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace,iT}}(0)$, $i = 1, \ldots, N$ be statistics based on the cross-sectionally independent data Y_{it}. Note that, equivalently, X_{it} may be considered, as the Saikkonen and Lütkepohl test is invariant to the values of the deterministic terms and hence these could be set to zero. According to Theorem 3.1 in Arsova and Örsal (2016), the asymptotic distribution of $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace,iT}}(0)$ is not only the same as the distribution of $\mathrm{LR}^{\mathrm{SL}}_{\mathrm{trace,iT}}(0)$, but the two statistics are also asymptotically equivalent. This implies the asymptotic independence of $\mathrm{LR}^{\mathrm{SL}*}_{\mathrm{trace,iT}}(0)$ over i and hence the independence of the corresponding p-values. □

As explained in Arsova and Örsal (2016), due to the defactoring procedure, the cointegrating matrix β_i cannot be estimated with the consistency rate $O_p(T^{-1})$. Therefore, the rank determination is carried out with a modified sequential testing procedure. By using a suitable estimator for the orthogonal complement[1] of the cointegrating matrix, i.e. $\hat{\beta}_{i\perp}$, it is possible to test for cointegrating rank higher than zero.

Within the modified sequential testing procedure, first the defactored data (i.e. Y_{it}^*) is tested for no cointegration. If $H_0 : r_i = 0, \forall i$ is rejected, then the next step is to test $H_0 : r_i = \bar{r} = 1$, where $\bar{r} = \max\{r_i | 1 \leq i \leq N\}$. For this purpose, the

[1] Let A be an $(m \times n)$ matrix with $\mathrm{rank}(A) = n$, then the orthogonal complement $A_\perp$ is an $(m \times (m-n))$ matrix with $\mathrm{rank}(A_\perp) = m - n$, such that $A_\perp' A = 0$.

orthogonal complement of the cointegrating space $\beta_{i\perp}$ is estimated from the defactored data. With the help of the estimator $\hat{\beta}_{i\perp}$ it is possible to select the appropriate candidates for stochastic trends in the system. In other words, the null hypothesis of higher cointegrating rank can be tested by checking whether $d = m - \bar{r}$ different stochastic trends exist. Therefore, the null of no cointegration is tested on the $d = m - \bar{r}$ dimensional vector $\hat{\beta}_{i\perp}' Y_{it}^*$. This procedure is repeated until the null hypothesis cannot be rejected or until $H_0 : \bar{r} = m - 1$ is tested.

The orthogonal complement of the cointegrating space $\beta_{i\perp}$ is estimated using the $\bar{r}$ largest eigenvalues of the eigenvalue problem

$$\left| \lambda \frac{1}{T} \hat{S}_{i,11} - \hat{S}_{i,10} \hat{S}_{i,00}^{-1} \hat{S}_{i,01} \right| = 0, \tag{11}$$

where the moment matrices $\hat{S}_{i,jk}$, $j, k \in \{0, 1\}$ are computed from the defactored data in the same way as in Johansen (1995, pp. 96–97) allowing for a deterministic trend.

4. Simulation study

4.1. Data generating process

The Monte Carlo study is based on the same DGP as in Arsova and Örsal (2016) in order to allow for comparison. The following three-variate DGP is used to generate the data:

$$Y_{it} = \mu_{0i} + \mu_{1i} t + X_{it} + \Lambda_i' F_t, \tag{12}$$

$$X_{it} = \begin{pmatrix} \psi_a & 0 & 0 \\ 0 & \psi_b & 0 \\ 0 & 0 & 1 \end{pmatrix} X_{it-1} + \varepsilon_{it}, \tag{13}$$

$$\varepsilon_{it} \sim N\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \theta_1 & \theta_2 \\ \theta_1 & 1 & \theta_3 \\ \theta_2 & \theta_3 & 1 \end{pmatrix} \right] \quad \text{and} \tag{14}$$

$$F_t = BF_{t-1} + u_t, \quad u_t \sim N(0, \Sigma_F), \tag{15}$$

where the terms θ_i, $i = 1, 2, 3$ induce instantaneous correlation between the stationary and the nonstationary components of the system. This process is a modification of the DGP used by Toda (1995), who finds that the performance of Johansen's (1995) likelihood-based cointegration tests depends on the magnitude of the instantaneous correlation coefficients. The same DGP is employed by Saikkonen and Lütkepohl (2000) with similar findings. In particular, the Saikkonen and Lütkepohl test has greater size and consequently better power when the correlation between the stationary and the nonstationary components is high. Within the simulation study we consider cases both with and without correlation between the stationary and nonstationary components. To save space only the simulation results with correlation are reported, as they represent the worse-case scenario in terms of size. Upon request simulation results without correlation can be provided.

Throughout the simulation study we use $T - 1 = \{25, 50, 100, 200, 500\}$ and $N = \{10, 25, 50, 100\}$. The initial values for X_{it} are set to zero. To generate DGP with true cointegrating rank zero, we set $\psi_a = \psi_b = 1$. The true cointegrating rank one is generated by the combinations $(\psi_a, \psi_b) = \{(0.7, 1), (0, 95, 1)\}$, and the true cointegrating rank two is generated by the combinations $(\psi_a, \psi_b) = \{(0.7, 0.7), (0.95, 0.7)\}$. The deterministic terms μ_{0i} and μ_{1i} are set to zero, since the LR-type test statistics of Saikkonen and Lütkepohl (2000) are invariant to the values of the deterministic terms. The number of common factors is $k = 2$ with $\sigma_F^2 = 1$. For non-stationary factors $B = I_2$, and for stationary ones $B = 0.9 I_2$. Finally, the factor loadings are independently uniformly distributed random variables with appropriate dimension, i.e. $\Lambda_i \sim i.i.d.\ U[-1, 3]$. Results with factor loadings $\Lambda_i \sim i.i.d.\ U[-1, 7]$ are similar and available upon request. The number of replications is set to 1000. The simulations are executed in GAUSS.

4.2. Simulation results

Table 1 presents the size results of the new tests for different experimental settings with different true cointegrating ranks. The left part of the table shows the size properties of the standardized Fisher-type test (P_N^*) and the right part of the table presents the results of the inverse normal test ($P_{\Phi^{-1}}^*$). Both tests have size distortions when the true cointegrating rank is zero and T is small, i.e. $T = 25$. Size distortions are also present when the cross-sectional dimension is higher than the time dimension as a result of the violation of the assumption that $T \to \infty$ first, followed by $N \to \infty$, or in case of joint limits, $N/T \to 0$. With increase in both the time and the cross-sectional dimension the size of both tests approaches the 5% nominal significance level. Overall, for the true cointegrating rank of zero the inverse normal test has better size properties when $T \geq 50$.

On the contrary, both tests are undersized when the true cointegrating rank is higher than zero. However, with increase in both T and N the size reaches the 5% nominal level, when the stationary process(es) in the system are not near unit root

Table 1
Size of the tests for different true cointegrating rank conditions.

		P_N^*					$P_{\Phi^{-1}}^*$				
		$r_0 = 0$	$r_0 = 1,\ \psi_b = 1$		$r_0 = 2,\ \psi_b = 0.7$		$r_0 = 0$	$r_0 = 1,\ \psi_b = 1$		$r_0 = 2,\ \psi_b = 0.7$	
T−1	N		$\psi_a = 0.7$	$\psi_a = 0.95$	$\psi_a = 0.7$	$\psi_a = 0.95$		$\psi_a = 0.7$	$\psi_a = 0.95$	$\psi_a = 0.7$	$\psi_a = 0.95$
25	10	0.118	0.012	0.002	0.000	0.000	0.089	0.017	0.001	0.000	0.000
	25	0.188	0.004	0.000	0.000	0.000	0.165	0.004	0.000	0.000	0.000
	50	0.219	0.001	0.000	0.000	0.000	0.218	0.004	0.000	0.000	0.000
	100	0.286	0.003	0.000	0.000	0.000	0.297	0.004	0.000	0.002	0.000
50	10	0.078	0.032	0.000	0.015	0.000	0.056	0.030	0.001	0.027	0.002
	25	0.092	0.032	0.001	0.012	0.000	0.068	0.021	0.002	0.025	0.001
	50	0.091	0.014	0.000	0.002	0.000	0.080	0.016	0.000	0.020	0.000
	100	0.136	0.006	0.000	0.002	0.000	0.128	0.007	0.000	0.015	0.000
100	10	0.061	0.038	0.006	0.034	0.007	0.056	0.023	0.005	0.025	0.017
	25	0.080	0.028	0.002	0.021	0.004	0.065	0.025	0.002	0.026	0.014
	50	0.069	0.022	0.000	0.024	0.001	0.066	0.020	0.000	0.031	0.013
	100	0.093	0.010	0.000	0.010	0.001	0.079	0.010	0.001	0.025	0.009
200	10	0.083	0.057	0.022	0.029	0.020	0.056	0.035	0.018	0.029	0.021
	25	0.072	0.028	0.006	0.028	0.007	0.054	0.018	0.005	0.032	0.017
	50	0.063	0.031	0.008	0.030	0.005	0.056	0.022	0.013	0.039	0.018
500	10	0.076	0.046	0.027	0.043	0.034	0.049	0.028	0.018	0.035	0.027
	25	0.062	0.037	0.019	0.047	0.024	0.050	0.038	0.018	0.045	0.029
	50	0.064	0.041	0.016	0.039	0.014	0.063	0.040	0.020	0.053	0.032

Notes: r_0 denotes the true cointegrating rank of the DGP. The results are based on the DGP which allows for correlation between the stationary and nonstationary components of the process. For the process with $r_0 = 0$, we set $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$, since the parameters θ_i, $i = 1, 2, 3$, show the correlation only between the stationary and nonstationary components. If $r_0 = 1$, then $(\theta_1, \theta_2, \theta_3) = (0.8, 0.3, 0)$, and if $r_0 = 2$, then $(\theta_1, \theta_2, \theta_3) = (0, 0.8, 0.3)$.

Table 2
Power of the tests when the hypothesized rank is below the true rank.

		P_N^*						$P_{\Phi^{-1}}^*$					
		$r_0 = 1,\ \psi_b = 1$		$r_0 = 2,\ \psi_b = 0.7$				$r_0 = 1,\ \psi_a = 1$		$r_0 = 2,\ \psi_b = 0.7$			
		$\psi_a = 0.7$	$\psi_a = 0.95$	$\psi_a = 0.7$		$\psi_a = 0.95$		$\psi_a = 0.7$	$\psi_a = 0.95$	$\psi_a = 0.7$		$\psi_a = 0.95$	
T-1	N	H(0)	H(0)	H(0)	H(1)	H(0)	H(1)	H(0)	H(0)	H(0)	H(1)	H(0)	H(1)
25	10	0.632	0.130	0.789	0.038	0.283	0.003	0.618	0.111	0.786	0.041	0.264	0.004
	25	0.960	0.190	0.992	0.044	0.520	0.004	0.972	0.173	0.993	0.085	0.540	0.007
	50	1	0.284	1	0.080	0.744	0.002	1	0.284	1	0.214	0.789	0.004
	100	1	0.402	1	0.112	0.935	0.000	1	0.420	1	0.353	0.965	0.007
50	10	0.995	0.143	0.999	0.565	0.739	0.025	0.995	0.119	0.999	0.642	0.757	0.040
	25	1	0.246	1	0.932	0.990	0.074	1	0.238	1	0.977	0.996	0.139
	50	1	0.404	1	1	1	0.099	1	0.426	1	1	1	0.272
	100	1	0.650	1	1	1	0.192	1	0.696	1	1	1	0.534
100	10	1	0.393	1	0.998	0.999	0.361	1	0.395	1	0.999	0.999	0.421
	25	1	0.798	1	1	1	0.768	1	0.830	1	1	1	0.862
	50	1	0.985	1	1	1	0.962	1	0.990	1	1	1	0.988
	100	1	1	1	1	1	1	1	1	1	1	1	1
	10	1	0.946	1	1	1	0.972	1	0.958	1	1	1	0.982
	25	1	1	1	1	1	1	1	1	1	1	1	1
	50	1	1	1	1	1	1	1	1	1	1	1	1
500	10	1	1	1	1	1	1	1	1	1	1	1	1
	25	1	1	1	1	1	1	1	1	1	1	1	1
	50	1	1	1	1	1	1	1	1	1	1	1	1

Notes: r_0 denotes the true cointegrating rank of the DGP. The results are based on the DGP which allows for correlation between the stationary and nonstationary components of the process. For the process with $r_0 = 0$, we set $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$, since the parameters θ_i, $i = 1, 2, 3$, show the correlation only between the stationary and nonstationary components. If $r_0 = 1$, then $(\theta_1, \theta_2, \theta_3) = (0.8, 0.3, 0)$, and if $r_0 = 2$, then $(\theta_1, \theta_2, \theta_3) = (0, 0.8, 0.3)$.

processes. When the underlying DGP has near unit root process, then the size of both tests is around 3% with increase in T and N. That means, higher T and N dimensions are necessary for the empirical size to reach the nominal size.

Table 2 shows the power results of the tests when the hypothesized rank is below the true cointegrating rank. H(0) and H(1) denote that the null hypothesis is rank zero and one, respectively. For both tests the power approaches quickly unity even in small samples when there is no near unit root process in the DGP. If there is a near unit root process then the tests cannot detect its presence very well for small T. With increase in both T and N the power also approaches unity even in the presence of a near unit root process. When the true cointegrating rank is two and the hypothesized rank is one, the inverse normal test has higher power in comparison to the standardized Fisher-type test mainly for $T \leq 100$.

Table 3
Size and power comparison of the P_N^* and $P_{\Phi^{-1}}^*$ tests with the P and P_m tests of Bai and Carrion-i-Silvestre (2013).

		Size, true rank $r_0 = 0$				Power, true rank $r_0 = 1$			
$T-1$	N	P_N^*	$P_{\Phi^{-1}}^*$	P	P_m	P_N^*	$P_{\Phi^{-1}}^*$	P	P_m
25	10	0.134	0.113	0.009	0.013	0.134	0.115	0.006	0.011
	25	0.168	0.149	0.000	0.000	0.178	0.161	0.000	0.000
	50	0.208	0.208	0.000	0.000	0.231	0.219	0.000	0.000
	100	0.298	0.297	0.000	0.000	0.300	0.337	0.000	0.000
50	10	0.091	0.065	0.018	0.026	0.106	0.089	0.026	0.033
	25	0.089	0.062	0.005	0.005	0.119	0.102	0.008	0.012
	50	0.098	0.084	0.000	0.000	0.151	0.150	0.000	0.000
	100	0.123	0.130	0.000	0.000	0.233	0.254	0.000	0.000
100	10	0.069	0.054	0.031	0.039	0.146	0.141	0.059	0.073
	25	0.084	0.057	0.012	0.016	0.254	0.237	0.053	0.062
	50	0.062	0.070	0.003	0.003	0.385	0.412	0.030	0.034
	100	0.070	0.075	0.000	0.000	0.573	0.664	0.008	0.009
200	10	0.074	0.068	0.037	0.043	0.430	0.433	0.125	0.155
	25	0.055	0.049	0.023	0.031	0.797	0.857	0.239	0.267
	50	0.069	0.053	0.013	0.017	0.983	0.994	0.318	0.351
500	10	0.055	0.042	0.058	0.074	0.991	0.990	0.379	0.420
	20	0.067	0.052	0.046	0.048	1	1	0.665	0.698
	50	0.074	0.068	0.028	0.034	1	1	0.908	0.919

Since we use the same DGP and the same simulation setup as in Arsova and Örsal (2016), we can compare their simulation results for the PSL^J_{def} test with our results. Note that the PSL^J_{def} test is a panel test based on the standardization of the average of the individual Saikkonen and Lütkepohl LR-type test statistics. For true cointegrating rank zero the $P^*_{\Phi^{-1}}$ test has slightly better size properties than the PSL^J_{def}, especially when N is small. The size of all the tests is almost equal when the true cointegrating rank of the system is one. The only difference is that the $P^*_{\Phi^{-1}}$ test is slightly more undersized than the other two tests when $T \geq 200$. The $P^*_{\Phi^{-1}}$ test has also better size properties in the presence a near unit root process for $r_0 = 2$ and $T \geq 100$.

Among all three tests P_N^* demonstrates the lowest power, whereas the $P^*_{\Phi^{-1}}$ test has the highest power for true cointegrating rank two and when the hypothesized rank is one. For the remaining simulation setups the power of the PSL^J_{def} and $P^*_{\Phi^{-1}}$ tests is comparable.

4.3. Comparison with the test of Bai and Carrion-i-Silvestre (2013)

In order to better align the newly proposed tests within the existing literature, we investigate their finite-sample properties alongside those of the no-cointegration tests of Bai and Carrion-i-Silvestre (2013). The latter authors as well assume that the cross-sectional dependence is driven by unobserved common factors, and allow these to be correlated with the stochastic regressors. They extract the dynamic factors and the idiosyncratic disturbances by an iterated procedure, and then determine the orders of integration of the common and idiosyncratic components separately. In the following simulations we focus on their Fisher-type test P and standardized Fisher-type test P_m, which combine the p-values of the individual modified Sargan–Bhargava (MSB) statistics based on the estimated idiosyncratic errors.

The DGP we employ for this purpose is the same as the one Bai and Carrion-i-Silvestre (2013) use for their simulation study; for brevity we refrain from presenting the details here and refer to the latter paper (Eqs. (5.1)–(5.6)).[2] We focus only on the time trend case with endogenous regressors and heterogeneous slope parameters, as in our view this setting is most empirically relevant. For this purpose we amend Eqs. (5.2) and (5.3) of Bai and Carrion-i-Silvestre's (2013) DGP to include linear time trend terms with slope coefficients set to 0.01 and −0.02, respectively. Size and power properties of the tests are investigated by setting the autoregressive parameter ρ_i for the idiosyncratic disturbance terms to be equal to 1 and 0.95, respectively (see Eq. (5.5) in Bai and Carrion-i-Silvestre, 2013). Therefore, the results presented in Table 3 below are comparable with those from their Table 4. To put all tests on equal grounds, the number of common factors is not estimated, but rather set equal to its true value. The number of lagged differences included in the estimation of the long-run variance for the individual MSB statistics and the lag order of the VAR approximations for the $P^*_{\Phi^{-1}}$ and P_N^* tests are 0 and 1, respectively. As before, panels with dimensions $T-1 \in \{25, 50, 100, 200, 500\}$ and $N \in \{10, 25, 50, 100\}$ are considered.

The size and power properties of the tests at the 5% level are presented in Table 3. The tests of Bai and Carrion-i-Silvestre (2013) are undersized for small and moderate T, with the size distortions diminishing as $T \to \infty$. Despite the different

[2] We are grateful to Josep Lluis Carrion-i-Silvestre for providing us with the GAUSS codes.

Table 4
Monetary exchange rate model: data description.

Variable	Description	Source
s	Nominal exchange rate per 1 USD; end-of-period	OECD
p	Consumer price index	OECD
y	Industrial production index	OECD
	Industrial production index for Switzerland	IMF IFS
p^T	Producer price index	IMF IFS
m	M3 for Japan	IMF IFS
	M2+ for Canada,	OECD
	M2 for Czech Republic, Norway, Poland	OECD
	M4 for Brazil, Turkey, UK	OECD
	M3 for the remaining countries	OECD, DANE for Colombia

DGP, the $P^*_{\Phi-1}$ and P^*_N tests display properties very similar to those reported in Table 1, being oversized for small T and approaching the correct size from above as T grows. As a consequence, they are also more powerful than Bai and Carrion-i-Silvestre's (2013) tests, which require large T. Therefore, in addition to being capable of not only detecting cointegration, but also of selecting the cointegrating rank, we would recommend the newly proposed $P^*_{\Phi-1}$ and P^*_N tests for practical use in view of their more desirable finite-sample properties.

5. Empirical illustration: monetary exchange rate model

To illustrate how the testing procedure works, we investigate the monetary exchange rate model (MERM). Building upon the assumptions of stable money demand functions, purchasing power parity and uncovered interest rate parity, the model postulates that the nominal exchange rate between two countries is determined by their relative money supply and output levels. From an econometric point of view such relationship can be established by testing for cointegration between the (logarithms of) the variables, and numerous studies analyzing this issue for different countries and by various techniques exist in the literature. The empirical evidence as to whether the MERM holds in practice is, however, inconclusive. Earlier single-country studies which have failed to find support for the model (e.g. Sarantis, 1994; Groen, 1999) have been criticized later on for using too short time span of data and low-power single-unit cointegration tests. Using panel data and newly developed panel cointegration techniques offers a convenient way to waive these criticisms. One of the earliest studies of the MERM employing panel data is that of Mark and Sul (2001), whose results based on data for 19 countries in the post-Bretton Woods era generally support the hypothesis of cointegration. Although Rapach and Wohar (2004) criticize the homogeneity restrictions imposed by Mark and Sul (2001), they as well find cointegration at the panel level using the same data set, but not on an individual country-by-country basis. The same data has been analyzed by Basher and Westerlund (2009), who, however, find that "the monetary model emerges only when the presence of structural breaks and cross-country dependence has been taken into account". Allowing only for cross-sectional dependence in their panel data for 8 Central and Eastern European countries, Dabrowski et al. (2014) find that the MERM holds. Hence, in view of the existing debate whether cointegration between nominal exchange rate and monetary fundamentals exists, we shed some light on the issue by applying the newly proposed cointegration tests. Our approach is closest to that of Carrion-i-Silvestre and Surdeanu (2011), who as well look at the stochastic properties of common and idiosyncratic components separately.

In the present analysis we work with the model of Dabrowski et al. (2014), which is:

$$s_{it} = \mu_{0i} + \mu_{1i}t + \beta_{i1}(m_{it} - m^*_{it}) + \beta_{i2}(y_{it} - y^*_t) + \beta_{i3}\Big[(p_{it} - p^T_{it}) - (p^*_t - p^{T*}_t)\Big] + u_{it}. \tag{16}$$

Here s_{it} is the natural logarithm of nominal exchange rate between country i and the USA, m_{it} is the natural logarithm of nominal money supply, y_{it} denotes the natural logarithm of industrial production index, p_{it} is the natural logarithm of consumer price index and the p^T_{it} is the natural logarithm of producer price index. The variables with asterisk represent the corresponding variables for the USA. We employ the tests using monthly data in the period January 1995–December 2007 for 19 countries: Brazil, Canada, Colombia, Czech Republic, Denmark, Hungary, India, Indonesia, Israel, Japan, Korea, Mexico, Norway, Poland, South Africa, Sweden, Switzerland, Turkey and the UK. A detailed description of the data can be found in Table 4.

Onatski's (2010) criterion selects eight common factors from the panel comprising all variables from all units. These eight factors are estimated after standardizing the first differenced and demeaned data, and explain almost 50% of their variation. The results of the panel cointegration tests applied on the estimated idiosyncratic components are summarized in Table 5. The cointegration test of Saikkonen and Lütkepohl (2000) is applied on the estimated idiosyncratic component of each country separately and the individual p-values are combined to form the panel statistics P^*_N and $P^*_{\Phi_{-1}}$ proposed in this study. According to the reported results, at the 5% significance level the P^*_N and $P^*_{\Phi-1}$ tests point to the existence of one cointegrating relation between the estimated idiosyncratic components. The PSL^J_{def} test of Arsova and Örsal (2016) as well detects the existence of a single cointegrating relation between the estimated idiosyncratic components.

Table 5
Monetary exchange rate model: results of the cointegration tests for the estimated idiosyncratic components.

		LR^{SL}_{trace} statistics				*p*-values			
Country	$\hat{\hat{p}}_i$	$r=0$	$r=1$	$r=2$	$r=3$	$r=0$	$r=1$	$r=2$	$r=3$
Brazil	2	41.61	13.96	8.66	3.62	0.114	0.829	0.473	0.256
Canada	2	43.89	19.82	6.25	0.41	0.070	0.404	0.747	0.940
Colombia	2	25.21	13.04	4.74	1.91	0.875	0.878	0.891	0.557
Czech Republic	2	29.24	17.66	7.71	0.58	0.690	0.565	0.580	0.900
Denmark	1	37.93	18.80	7.76	2.45	0.230	0.478	0.575	0.442
Hungary	2	32.37	16.94	8.40	0.86	0.510	0.621	0.502	0.828
India	2	24.85	14.80	6.28	1.89	0.888	0.776	0.744	0.563
Indonesia	4	26.91	12.64	4.21	1.69	0.807	0.897	0.928	0.612
Israel	2	36.28	21.68	6.22	0.56	0.302	0.284	0.751	0.906
Japan	3	28.15	15.40	6.97	1.97	0.747	0.736	0.667	0.543
Korea	2	57.47	13.82	7.83	2.82	0.002	0.837	0.566	0.374
Mexico	2	29.00	20.60	6.64	0.68	0.703	0.351	0.704	0.875
Norway	2	43.77	19.63	6.72	1.37	0.072	0.417	0.696	0.694
Poland	2	60.46	28.64	6.62	0.56	0.001	0.049	0.707	0.907
South Africa	2	21.30	10.05	6.87	4.73	0.969	0.975	0.678	0.147
Sweden	1	32.13	7.15	3.39	0.78	0.524	0.998	0.969	0.851
Switzerland	1	28.42	13.68	3.41	1.70	0.733	0.845	0.968	0.609
Turkey	2	48.69	27.14	14.04	0.33	0.021	0.075	0.094	0.958
UK	2	50.25	24.44	4.29	3.03	0.014	0.153	0.923	0.338
		Panel cointegration test statistics							
PSL		2.31**	−1.35	−2.64	−2.10				
P^*_N		3.74***	−0.98	−2.40	−2.02				
$P^*_{\Phi^{-1}}$		−1.91**	1.49	2.64	2.12				

Notes: r denotes the cointegrating rank under the null hypothesis. Eight common factors were extracted from the whole panel to estimate the idiosyncratic components. The cointegration tests are performed by including a linear time trend in the data generating process. The lag order $\hat{\hat{p}}_i$ of the VAR processes is determined by the Akaike Information Criterion ($p_{max} = 4$). *** and ** denote significance at the 1% level and 5% level, respectively.

Table 6
Monetary exchange rate model: results of the cointegration tests for the estimated factors.

	SL test		Johansen test	
r_0	LR^{SL}_{trace}	*p*-value	LR^{J}_{trace}	*p*-value
0	202.45	0.000***	251.61	0.000***
1	116.54	0.080	177.37	0.001***
2	85.13	0.125	119.95	0.034**
3	47.89	0.622	81.59	0.147
4	27.17	0.794	50.87	0.379
5	14.74	0.784	31.4	0.427
6	3.72	0.955	16.87	0.433
7	1.41	0.684	7.16	0.338

Notes: r denotes the cointegrating rank under the null hypothesis. The tests were performed with JMulTi. The lag order of the VAR process ($p = 2$) is determined by the Akaike Information Criterion. *** and ** denote significance at the 1% level and 5% level, respectively.

As a next step we investigate the stochastic properties of the eight common factors by applying the GLS-based LR-type trace test of Saikkonen and Lütkepohl (2000) and the LR trace test of Johansen (1995). The results are presented in Table 6. These tests point to the existence of one or three cointegrating relationships among the eight factors, respectively. This implies that their nonstationarity is driven by at least five global stochastic trends. These results are remarkably similar to the results of Carrion-i-Silvestre and Surdeanu (2011) who as well find a single cointegrating relationship between the idiosyncratic components of exchange rates and macroeconomic fundamentals, after extracting four nonstationary common factors.

Figs. 1–3 depict the estimated common components (i.e. $\hat{\Lambda}_i'\hat{F}_t$) along with the observed relative variables. In these plots both the observed data and the estimated common components have been detrended by OLS for the ease of comparison. This serves as an evidence that the relative importance of the common factors in determining the stochastic properties of the observed individual time series is high. The estimated eight common factors are presented in Fig. 4.

We therefore conclude that the four observed relative variables involved in the monetary exchange model for these 19 countries are mainly driven by at least five distinct global stochastic trends and the cointegrating relation between the estimated idiosyncratic components.

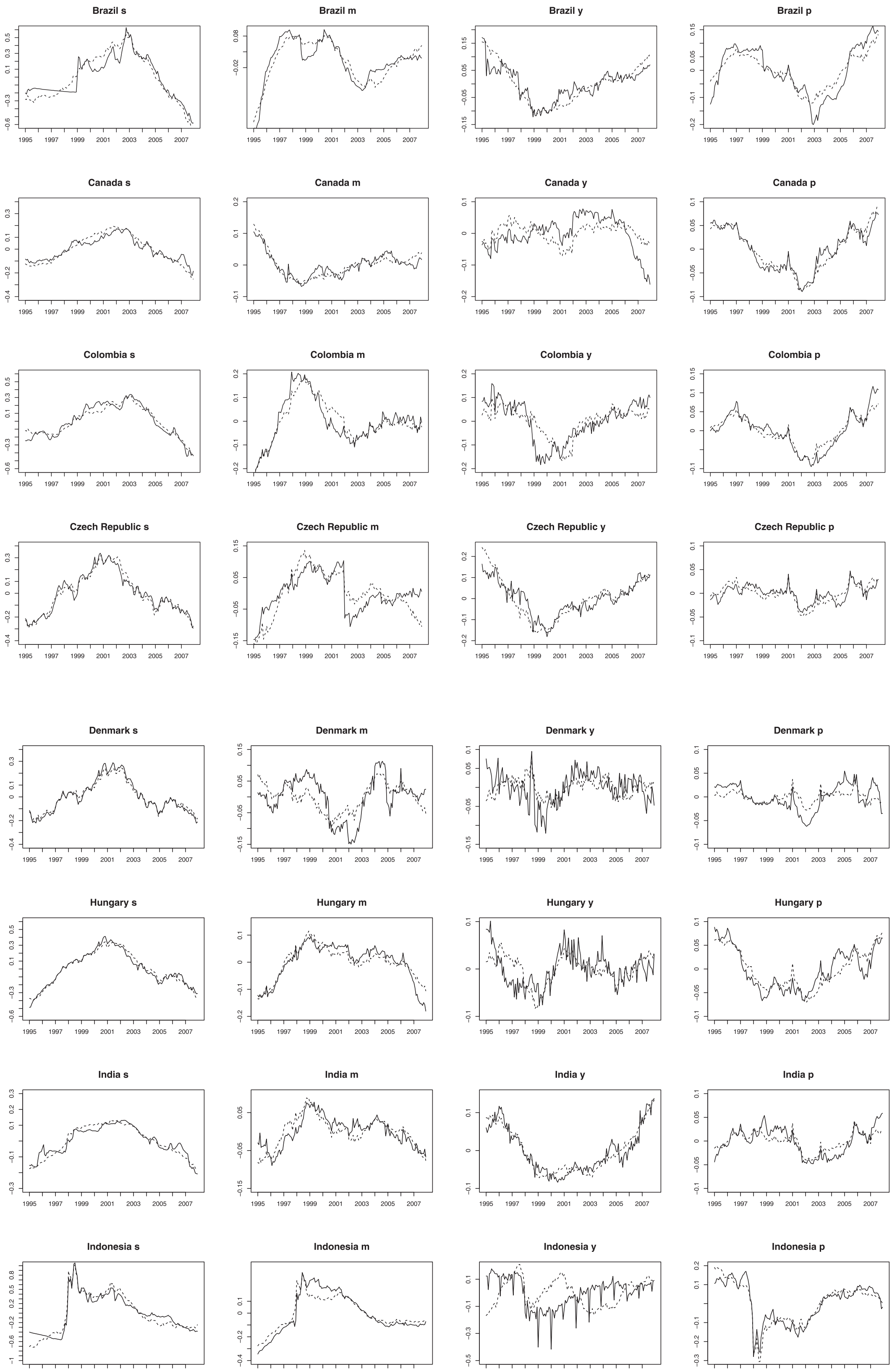

Fig. 1. OLS-detrended observed data (solid line) and OLS-detrended estimated common components (dashed line).

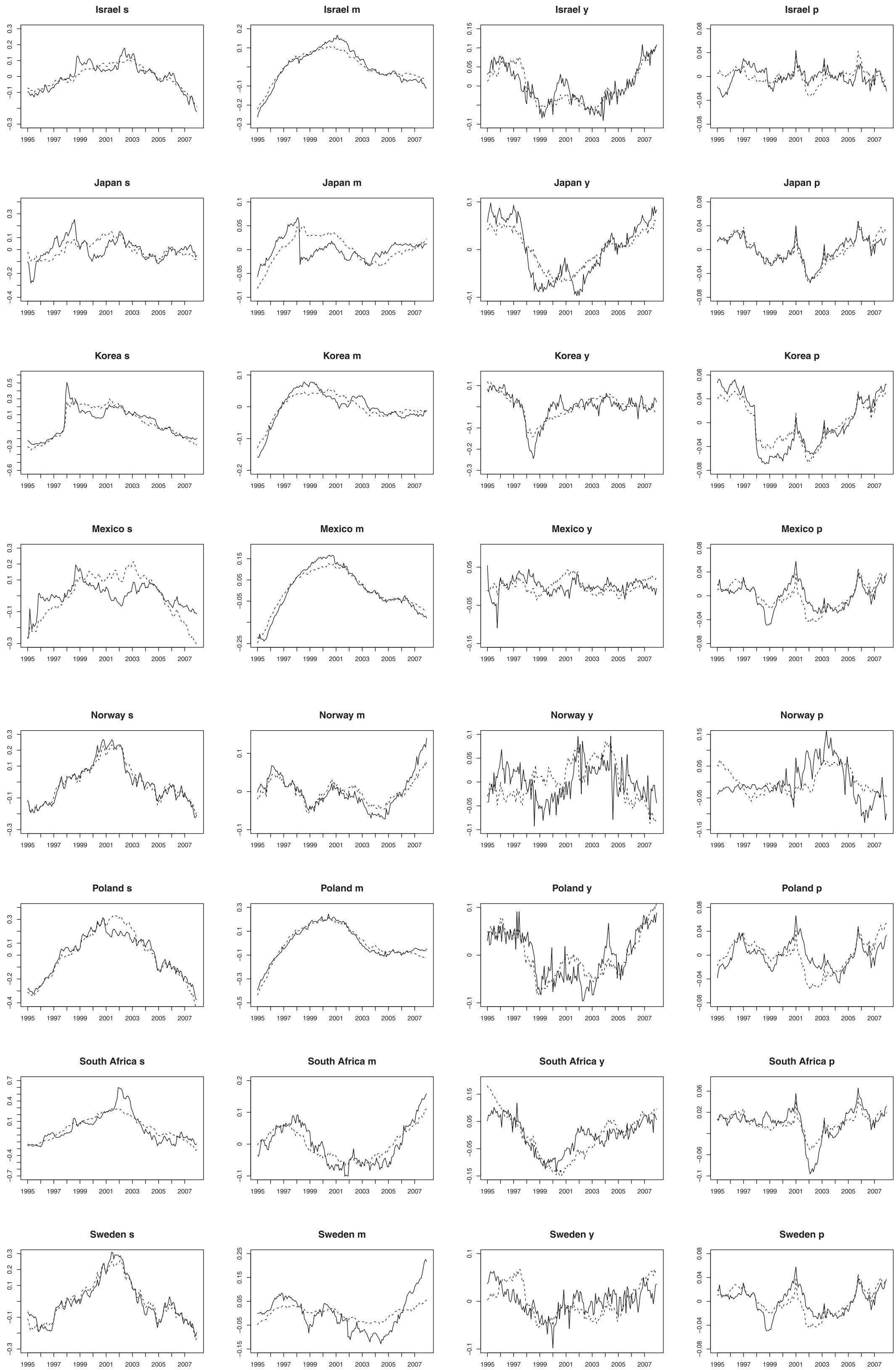

Fig. 2. OLS-detrended observed data (solid line) and OLS-detrended estimated common components (dashed line).

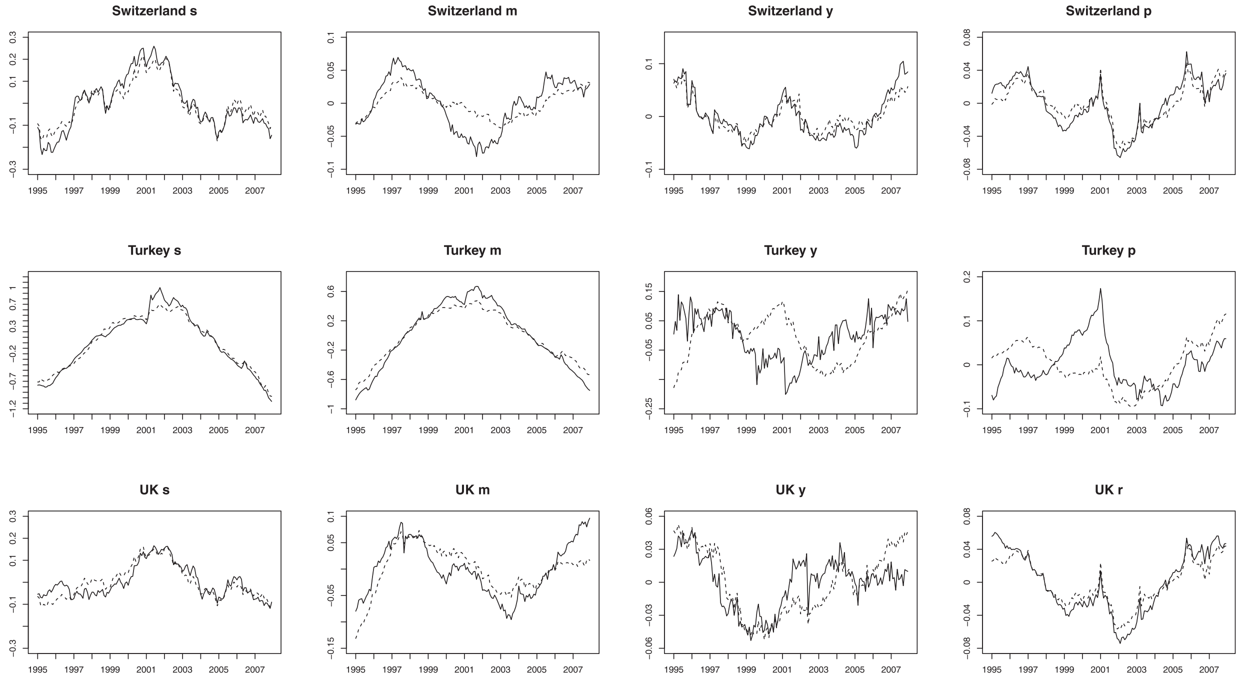

Fig. 3. OLS-detrended observed data (solid line) and OLS-detrended estimated common components (dashed line).

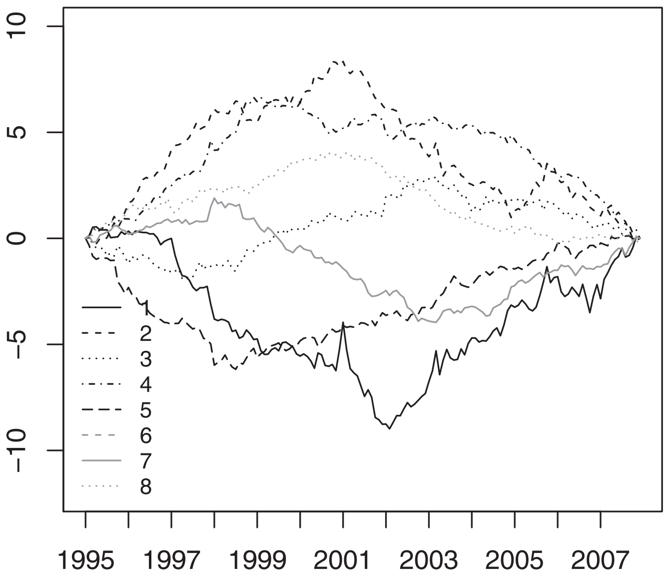

Fig. 4. Estimated common factors.

6. Conclusions

This paper makes use of a common factor framework and a meta analytic approach to propose new panel cointegrating rank tests. The tests are based on *p*-values combination of the individual LR-type test statistics of Saikkonen and Lütkepohl (2000). The testing procedure allows to test the idiosyncratic components and the common factors separately for cointegration. This useful approach allows to find out the main driving sources of the long-run stationary relations. The Monte Carlo study shows that the proposed tests have reasonable finite-sample properties—the power of the tests is high even when the time dimension of the panel is small. A comparison of the P_N^* and $P_{\Phi_{-1}}^*$ tests with the PSL_{def}^J test of Arsova and Örsal (2016) leads to the conclusion that the $P_{\Phi_{-1}}^*$ has preferable better finite-sample properties in some cases. Furthermore, the new tests are more powerful than the no-cointegration tests of Bai and Carrion-i-Silvestre (2013). Based on this we recommend the use of the newly proposed tests for empirical analysis.

Acknowledgment

Financial support by the German Research Foundation (DFG) through the project KA3145/1-2 is gratefully acknowledged.

Supplementary material

Supplementary material associated with this article can be found in the online version at 10.1016/j.ecosta.2016.10.001.

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