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Managing increasing environmental risks through agro-biodiversity and agri-environmental policies

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Abstract. Agro-biodiversity can provide natural insurance to risk-averse farmers by reducing the variance of crop yield, and to society at large by reducing the uncertainty in the provision of public-good ecosystem services such as e.g. CO\textsubscript{2} storage. We analyze the choice of agro-biodiversity by risk-averse farmers who have access to financial insurance, and study the implications for agri-environmental policy design when on-farm agro-biodiversity generates a positive risk externality. While increasing environmental risk leads private farmers to increase their level of on-farm agro-biodiversity, the level of agro-biodiversity in the laissez-faire equilibrium remains inefficiently low. We show how either one of two agri-environmental policy instruments can cure this risk-related market failure: an ex-ante Pigouvian subsidy on on-farm agro-biodiversity and an ex-post compensation payment for the actual provision of public environmental benefits. In the absence of regulation, welfare may increase rather than decrease with increasing environmental risk, if the agro-ecosystems is characterized by a high natural insurance function, low costs and large external benefits of agro-biodiversity.

JEL-Classification: Q1, Q57, H23, D62

Keywords: agro-biodiversity, ecosystem services, agri-environmental policy, insurance, risk-aversion, uncertainty

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1 Introduction

While private farmers manage agro-ecosystems primarily for the direct ecosystem services they provide (e.g. crop yield), it is by now widely acknowledged that agro-ecosystems provide numerous ecosystem services as joint products, including e.g. the regulation of pests, diseases, water runoff, CO$_2$ storage, or landscape conservation (OECD 2001, Heal and Small 2002). Typically, these regulating and cultural services (Millennium Ecosystem Assessment 2005) have the characteristics of public goods. Both private and public ecosystem services are strongly influenced by on-farm agrobiodiversity.

One important dimension of the use of agro-biodiversity by farmers, and the private and public benefits associated with it, is the risk-dimension. The management of various risks is traditionally one of the main challenges in agriculture. Farmers face a wide variety of production and marketing risks, including stochasticities in weather, pests, diseases or market prices. As a result, farming income is highly uncertain. Two major strategies for risk-averse farmers to hedge their income risk are (i) to grow a diverse portfolio of crop species and varieties as a form of natural insurance and (ii) to buy financial insurance.$^1$ While (i) is a very traditional low-tech and low-capital strategy that is still widely used in many regions of the world where financial and insurance markets are not existent or not yet well developed, strategy (ii) is growing in importance as farmers have better access to financial and insurance markets and financial and insurance services are being developed specifically for farmers, such as e.g. crop yield insurance or weather index insurance schemes (World Bank 2005).

With global environmental change, environmental risks are increasing (IPCC 2007, Millennium Ecosystem Assessment 2005, UNEP 2007). For example, in many regions of the world the statistical distribution of rainfall, storms or temperature is spreading due to global climate change, leading to a higher variance and to a higher number of extreme events. Also, the ecological risks of pests and diseases are increasing due to an increase in the introduction of alien species.

In this paper, we study how such increasing environmental risks can be managed by farmers through on-farm agro-biodiversity and financial insurance from the market. We analyze the implications for individually and socially optimal agrobiodiversity management and policy design when on-farm agro-biodiversity generates a positive externality on society at large in terms of positively influencing the

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$^1$A third strategy, which is not explicitly considered here, is the use of risk-reducing inputs into the agricultural production process, such as e.g. irrigation, fertilizer, pesticides, or enhanced (through breeding or genetic modification) varieties.
There is broad evidence in economics and ecology that agro-biodiversity has a natural insurance function concerning both private and public agro-ecosystem services. Several empirical studies have shown that higher agro-biodiversity may increase the mean level, and decrease the variance, of crop yields and farm income (Smale et al. 1998, Schläpfer et al. 2002, Widowsky and Rozelle 1998, Zhu et al. 2000, Di Falco and Perrings 2003, 2005, Di Falco et al. 2007). This result is also supported by recent theoretical, experimental and observational research in ecology about the role of biodiversity for the provision of ecosystem services (Hooper et al. 2005, Kinzig et al. 2002, Loreau et al. 2001, 2002). It has been conjectured that risk-averse farmers use crop diversity in order to hedge their income or consumption risk (Birol et al. 2006a, 2006b, Di Falco and Perrings 2003). Since agro-biodiversity provides natural insurance to risk-averse farmers, they tend to employ a higher level of agro-biodiversity in the face of uncertainty (Baumgärtner 2007, Quaas and Baumgärtner 2008).

Instead of making use of natural insurance, farmers can also buy financial insurance to hedge their income risk. Since agro-biodiversity as a form of natural insurance and financial insurance from the market are substitutes for an individual risk-averse farmer, improved access to the latter drives out the former (Ehrlich and Becker 1972, Baumgärtner 2007, Quaas and Baumgärtner 2008). Indeed, the extent to which farmers rely on agro-biodiversity as a natural insurance is found to be affected by agricultural policies such as subsidized crop yield insurance or direct financial assistance (Di Falco and Perrings, 2005). Some studies have shown that financial insurance tends to have ecologically negative effects. Horowitz and Lichtenberg (1993, 1994a, 1994b) show that financially insured farmers are likely to undertake riskier production – with higher nitrogen and pesticide use – than uninsured farmers do. A similar result is pointed out by Mahul (2001), assuming a weather-based insurance.

In the trade-off between financial insurance and natural insurance through agrobiodiversity, a market failure problem arises from the fact that agro-biodiversity does not only provide private on-farm benefits, but also gives rise to public benefits. As a general result, the privately determined level of on-farm agro-biodiversity is lower than the socially optimal one (Heal et al. 2004). In particular, such market failure stems from the risk-changing characteristics of agro-biodiversity and risk-averse behavior of private farmers (Baumgärtner 2007, Quaas and Baumgärtner 2008).

The literature on the provision of a public good under uncertainty suggests that
private uncertainty and risk-aversion increase the efficiency of the private provision of public goods (Bramoullé and Treich 2005, Sandler and Sterbenz 1990, Sandler et al. 1987). The focus in this literature is on the properties of the utility function, while the production of the public good (or public bad) is typically modelled in a trivial way, i.e. one unit of money spent on providing the public good equals one unit of the public good provided. In this paper, we study this issue in a more realistic settings in which the production of a public good – such as agro-biodiversity’s public insurance function – is generated in a complex system – such as a multi-scale ecological-economic system.

This paper goes beyond existing studies in three respects: (i) the agro-ecosystem is modelled in a more general manner; (ii) the focus here is on the question of how increasing environmental risks affect the trade-off between natural and financial insurance as well as the underprovision of agro-biodiversity and social welfare; (iii) we explicitly study the policy implications for regulating agricultural production, thereby distinguishing between ex-ante and ex-post policy instruments under uncertainty.

Our analysis is based on a stylized model. Crop yield on an individual farm is random because of exogenous sources of environmental risk (e.g. weather, diseases or pests); its statistical distribution (mean and variance) is determined by the level of agro-biodiversity, and the variance may be increasing. The level of on-farm agro-biodiversity not only determines the distribution of farm income, but also generates an external benefit to society at large in terms of a reduced risk in the provision of public-good ecosystem services such as the regulation of pests, diseases, water runoff, or CO$_2$ storage. The farmer is risk-averse and chooses the level of on-farm agro-biodiversity so as to maximize the expected utility of farm income. When making this choice, he has also access to financial income insurance.

We show that increasing environmental risk leads private farmers to increase their level of on-farm agro-biodiversity. Yet, the privately determined level of on-farm agro-biodiversity is inefficiently low. We show that an ex-ante Pigouvian subsidy on on-farm agro-biodiversity can cure this market failure problem. The subsidy rate increases with public risk and decreases with private risk. Likewise, an ex-post compensation payment for the actual provision of public environmental benefits can cure this market failure problem. We show that, if the individual farmer is more risk-averse than society at large, the compensation payment should be smaller than under certainty. If the market failure problem is not optimally regulated the welfare effect of increasing environmental risk is ambiguous. We show that for agro-ecosystems with a high natural insurance function, low costs and large external
benefits of agro-biodiversity, welfare in the absence of regulation increases rather than decreases with increasing environmental risk.

The paper is organized as follows. In Section 2, we specify the model. The analysis and results are presented in Section 3, with all proofs and formal derivations contained in the Appendix. Section 4 discusses the results and concludes.

2 Model

We consider a farmer who manages an agro-ecosystem for the service, i.e. crop yield, it provides. Due to stochastic fluctuations in environmental conditions the provision of the agro-ecosystem service is uncertain. Its statistical distribution depends on the state of the agro-ecosystem in terms of agro-biodiversity, which is determined by the farmer’s management decision. As a result, the statistical distribution of agro-ecosystem service and, hence, of income depend on ecosystem management. At the same time, agro-biodiversity determines the statistical distribution of ecosystem services that accrue to society at large, i.e. to public-good ecosystem services. We capture these relationships in a stylized model as follows.

2.1 Agro-ecosystem management

The farmer chooses a level $v$ of agro-biodiversity by selecting a portfolio of different crop varieties. Given the level of agro-biodiversity $v$, the agro-ecosystem provides the farmer with the desired service, i.e. total crop yield, at a level $s$ which is random. For simplicity we assume that the agro-ecosystem service directly translates into monetary income and that its mean level $E_s = \mu$ is independent of the level of agro-biodiversity and constant.\(^2\) The variance of agro-ecosystem service depends on the level of agro-biodiversity $v$ as follows

$$\text{var } s = \theta \sigma^2(v) \quad \text{where} \quad \sigma^2(v) < 0 \text{ and } \sigma^2''(v) \geq 0 . \quad (1)$$

An increase in the parameter $\theta > 0$ models a mean-preserving spread of risk (Rothschild and Stiglitz 1970). This allows us to discuss the private effects of increased environmental uncertainty in a convenient way. For illustrative purpose, we will consider the following specific example:

$$\sigma^2(v) = \sigma_0 v^{1-\eta} \quad \text{with} \quad \eta > 1 . \quad (2)$$

\(^2\)Empirical evidence suggests that $\mu$ may depend on $v$ (see Section 1). We explored the impact of such relationships in previous versions of the model. Here, we neglect such a dependence of $\mu$ on $v$ as it complicates the analysis while not adding further insights into the insurance dimension of the issue under study.
The constant $\eta$ parameterizes the natural insurance capacity of the agro-ecosystem: the larger $\eta$, the stronger does the variance of agro-ecosystem service (total crop yield) decline with the level of agro-biodiversity.

### 2.2 Financial insurance

In order to analyze the influence of availability of financial insurance on the farmers’ choice of agro-biodiversity, we introduce financial insurance in a simple and stylized way. We assume that the farmer has the option of buying financial insurance under the following contract: (i) The farmer chooses the fraction $a \in [0, 1]$ of insurance coverage. (ii) He receives (pays) $a (E_s - s)$ from (to) the insurance company as an actuarially fair indemnification benefit (insurance premium) if his realized income is below (above) the mean income. In order to abstract from any problems related to informational asymmetry, we assume that the statistical distribution as well as the actual level $s$ of agro-ecosystem service are observable to both insurant and insurance company. (iii) In addition to (3), the farmer pays the transaction costs of insurance. The costs of insurance over and above the actuarially fair insurance premium, which are a measure of the real costs of insurance to the farmer, are assumed to follow the cost function

$$\delta a \operatorname{var} s,$$

where the parameter $\delta \geq 0$ describes how actuarially unfair is the insurance contract. The costs increase linearly with the insured part of income variance. This captures in the simplest way the idea that the costs of insurance increase with the extent of insurance.

### 2.3 Farmer’s income, preferences and decision

The farmer chooses the level of agro-biodiversity $v$ and financial insurance coverage $a$. A higher level of agro-biodiversity carries costs $c > 0$ per unit of agro-

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3For a formal motivation in terms of agro-biodiversity’s insurance value, see Section 3.1

4This benefit/premium-scheme is actuarially fair, because the insurance company has an expected net payment stream of $E[a(E_s - s)] = 0$. This model of insurance is fully equivalent to the traditional model of insurance (e.g. Ehrlich and Becker 1972: 627) where losses compared with the maximum income are insured against and the insurant pays a constant insurance premium irrespective of actual income. In this traditional model, the net payment would exactly amount to (3); for a formal proof see Quaas and Baumgärtner (2008: Appendix A.1).
biodiversity. These costs may be due to increased cropping, harvesting and marketing effort, and are purely private. Adding up income components, the farmer’s (random) income $y$ is given by

$$y = (1 - a) s - cv + a \mathcal{E}s - \delta a \text{var } s .$$ \hspace{1cm} (5)

Since the agro-ecosystem service $s$ is a random variable, net income $y$ is a random variable, too. The uncertain part of income is captured by the first term in Equation (5), while the other components are certain. Obviously, increasing $a$ to one allows the farmer to reduce the uncertain income component down to zero.

The mean $\mathcal{E}y$ and the variance $\text{var } y$ of the farmer’s income $y$ are determined by the mean and variance of agro-ecosystem service, which depends on the level of agro-biodiversity (Equation 1),

$$\mathcal{E}y = \mu - cv - \delta a \theta \sigma^2(v) \quad \text{and}$$

$$\text{var } y = (1 - a)^2 \theta \sigma^2(v) .$$ \hspace{1cm} (6)

Mean income is given by the mean level of agro-ecosystem service $\mu$, minus the costs of agro-biodiversity $cv$ and the costs of financial insurance $\delta a \theta \sigma^2(v)$. For an actuarially fair financial insurance contract ($\delta = 0$), mean income equals mean net income from agro-ecosystem use, $\mu - cv$. The variance of income vanishes for full financial insurance coverage, $a = 1$, and equals the full variance of agro-ecosystem service, $\theta \sigma^2(v)$, without any financial insurance coverage, $a = 0$.

The farmer is assumed to be non-satiated and risk-averse with respect to his uncertain income $y$. There exists empirical evidence on how agro-biodiversity influences the mean and variance of agro-ecosystem services, but hardly on the full statistical distribution. This restricts the class of risk preferences which can meaningfully be represented in our model to utility functions which depend only on the first and second moment of the probability distribution, i.e. on the mean and the variance. Specifically, we assume the following expected utility function, where $\rho > 0$ is a parameter describing the farmer’s degree of risk aversion (Arrow 1965, Pratt 1964):\(^5\)

$$U = \mathcal{E}y - \frac{\rho}{2} \text{var } y .$$ \hspace{1cm} (8)

\section*{2.4 External benefits of agro-biodiversity}

The agro-ecosystem does not only provide the private ecosystem service crop-yield, but also ecosystem services that have the characteristics of a public good, such as

\(^5\)More general utility functions of the mean-variance type would complicate the analysis without generating further insights.
e.g. regulation of pests, diseases, water runoff, or CO₂ storage. Since these ecosystem services depend on agro-biodiversity, the farmer’s private decision on the level of agro-biodiversity \( v \) affects not only his private income risk, as expressed by the variance of on-farm agro-ecosystem service, \( \text{var} s \) (Equation 1), but also causes external effects.

Let \( B(v) \) capture all benefits of public-good ecosystem services that depend on on-farm agro-biodiversity \( v \). In particular, we assume that an external benefit of on-farm agro-biodiversity arises, as the uncertainty in the provision of public ecosystem services is reduced by a higher level of agro-biodiversity.

\[
E_B(v) = \gamma
\]
\[
\text{var} B(v) = \Theta \Sigma^2(v) \quad \text{where} \quad \Sigma^2'(v) < 0 \text{ and } \Sigma^2''(v) \geq 0 .
\]

For simplicity we assume – as in the case of the private ecosystem service – that the mean level of the public ecosystem service is independent of on-farm biodiversity \( v \). By contrast, the variance of the public ecosystem service decreases with \( v \), capturing a natural insurance function of agro-biodiversity also for the public ecosystem service. An increase in the parameter \( \Theta > 0 \) models a mean-preserving spread of risk (Rothschild and Stiglitz 1970). This allows us to discuss the public effects of increased uncertainty in a convenient way. The external welfare effect of on-farm agro-biodiversity is

\[
E_B - \Omega \text{var} B ,
\]

where \( \Omega > 0 \) is a parameter describing the degree of social risk aversion. Furthermore, we assume that the private and the public risks associated with \( v \) are uncorrelated, as they are associated with different types of ecosystem services. The total (i.e. private plus external) welfare effect of on-farm agro-biodiversity, thus, is:\(^6\)

\[
W = E_y + E_B - \frac{\rho}{2} \text{var} y - \frac{\Omega}{2} \text{var} B .
\]

3 Analysis and results

The analysis proceeds in four steps: First, we identify agro-biodiversity’s private and public insurance value (Section 3.1). Next, we discuss the laissez-faire allocation which arises if the farmer individually maximizes expected utility from farm income (Section 3.2). Then, we study the efficient allocation which is obtained by

\(^6\)In case of correlated private and public risks Equation (12) would generalize to \( W = E_y + E_B - \frac{\rho}{2} \text{var} y - \frac{\gamma}{2} \text{var} B - \gamma \text{cov}(y, B) .\)
maximizing social welfare (Section 3.3). Finally, we investigate how policy measures to internalize the externalities and welfare are influenced by increasing private and public environmental risks, as described by the parameters $\theta$ and $\Theta$ (Section 3.4).

### 3.1 The insurance value of agro-biodiversity

In order to precisely define the insurance value of agro-biodiversity, recall that by choosing the level of agro-biodiversity $v$ and the fraction of financial insurance coverage $a$ the farmer actually chooses a particular income lottery, which in our model is characterized by the mean $\mathbb{E}y = \mu - cv - \delta a \theta \sigma^2(v)$ and variance $\text{var } y = (1 - a)^2 \theta \sigma^2(v)$ of income (Equations 6, 7). These are determined by $v$ and $a$ and, therefore, one may speak of ‘the lottery $(v, a)$’.

One standard method of valuing the riskiness of a lottery to a decision maker is to calculate the risk premium $R$ of the lottery, which is defined as the amount of money that leaves the decision maker equally well off, in terms of utility, between the two situations of (i) receiving for sure the expected pay-off from the lottery $\mathbb{E}y$ minus the risk premium $R$, and (ii) playing the risky lottery with random pay-off $y$ (e.g. Dasgupta and Heal 1979: 381, Kreps 1990: 84). With utility function (8), the risk premium $R$ of a lottery with mean pay-off $\mathbb{E}y$ and variance $\text{var } y$ is simply given by:

$$R = \frac{\rho}{2} \text{var } y .$$

In the model employed here the risk premium of the farmer’s income lottery thus depends on the levels of agro-biodiversity $v$ and of financial insurance coverage $a$: $R(v, a) = \frac{\rho}{2} (1 - a)^2 \theta \sigma^2(v) .

The insurance value of agro-biodiversity can now be defined based on the risk premium of the lottery $(v, a)$: The insurance value $V^v$ of agro-biodiversity $v$ is given by the change of the risk premium $R$ of the lottery $(v, a)$ due to a marginal change in the level of agro-biodiversity $v$:

$$V^v(v, a) := -\frac{\partial R(v, a)}{\partial v} .$$

Thus, the insurance value of agro-biodiversity is the marginal value of agro-biodiversity in its function to reduce the risk premium of the farmer’s income risk from harvesting uncertain agro-ecosystem services. Being a marginal value, it depends on the existing level of agro-biodiversity $v$. It also depends on the actual level of financial insurance coverage $a$. The minus sign in the defining Equation (15) serves to express agro-biodiversity’s ability to reduce the risk premium of the lottery $(v, a)$ as a
positive value. Applying Definition (15) to Equation (14), one obtains the following insurance value $V^v(v,a)$ of agro-biodiversity in this model.

$$V^v(v,a) = -\frac{\rho}{2} (1-a)^2 \theta \sigma^2(v) > 0 .$$

From Equation (16) it is apparent that the insurance value of agro-biodiversity has an objective, a subjective and an institutional dimension. The objective dimension is captured by the sensitivity of the variance of agro-ecosystem services to changes in agro-biodiversity, $\theta \sigma^2$; the subjective dimension is captured by the farmer’s degree of risk aversion, $\rho$; and the institutional dimension is captured by the farmer’s extent of financial insurance coverage, $a$, which depends on institutional conditions (see below). The insurance value of agro-biodiversity $V^v$ increases with the sensitivity of the variance of agro-ecosystem services to changes in agro-biodiversity, $|\theta \sigma^2|$, and with the degree $\rho$ of the farmer’s risk aversion. It decreases with the farmer’s extent of financial insurance coverage, $a$. In the extreme, for vanishing subjective risk-aversion, $\rho = 0$, or for full financial insurance coverage, $a = 1$, agro-biodiversity’s insurance value vanishes. As a function of the level $v$ of agro-biodiversity, the insurance value $V^v(v,a)$ decreases: as agro-biodiversity becomes more abundant (scarcer), its insurance value decreases (increases).

In the example of specification (2), agro-biodiversity’s insurance value $V^v(v,a)$ is isoelastic with respect to changes in the level of agro-biodiversity $v$, and $\eta$ expresses this elasticity.° That is, an increase of agro-biodiversity by 1% always leads to an increase of its insurance value by $\eta$%. This motivates the interpretation of $\eta$ as the agro-ecosystem’s natural insurance capacity.

One can also define the insurance value of financial insurance as

$$V^a(v,a) := -\frac{\partial R(v,a)}{\partial a} .$$

With Expression (14) for the risk premium of the income lottery $(v,a)$, the insurance value $V^a(v,a)$ of financial insurance is thus given by

$$V^a(v,a) = \rho (1-a) \theta \sigma^2(v) .$$

Similar to the insurance value of agro-biodiversity the insurance value of financial insurance can be interpreted in terms of an objective, a subjective and an institutional dimension.

So far, we have been discussing agro-biodiversity’s private insurance value to an individual farmer, based on the private risk premium $R(v,a)$ (Equation 14) of the 

°Formally, $-v \frac{\partial V^v(v,a)}{\partial v} / V^v(v,a) \equiv \eta$. 

10
farmer’s private income lottery. Beyond that, agro-biodiversity also has a *public* insurance value. On-farm agro-biodiversity has an additional risk-reducing value due to its external benefit (11), i.e. there exists a public risk premium,

\[ R_{\text{pub}}(v) = \frac{\Omega}{2} \text{var } B = \frac{\Omega}{2} \Theta \Sigma^2(v) , \]  

(19)

which is in addition to the private one, giving rise to a public insurance value of

\[ V_{\text{pub}}(v) = -\frac{\partial R_{\text{pub}}(v)}{\partial v} = -\frac{\Omega}{2} \Theta \Sigma^2(v) > 0 . \]  

(20)

The total insurance value of on-farm agro-biodiversity then is the sum of the private and the public insurance value. Similar to the private insurance value of agro-biodiversity, the public insurance value depends on the properties of the agro-ecosystem. In particular it depends on how agro-biodiversity reduces the risk in the provision of the public ecosystem service. Also, the public insurance value increases with the degree \( \Omega \) of social risk aversion.

### 3.2 Laissez-faire allocation

As laissez-faire allocation \((v^*, a^*)\) we consider the allocation in which the farmer individually chooses the level of agro-biodiversity \(v\) and financial insurance coverage \(a\) such as to maximize expected utility (Equation 8) subject to constraints (6) and (7). Formally, the farmer’s decision problem is

\[
\max_{v, a} U = \mu - cv - d a \theta \sigma^2(v) - \frac{\rho}{2} (1 - a)^2 \theta \sigma^2(v) . 
\]  

(21)

The laissez-faire allocation has the following properties.

**Proposition 1**

An (interior) laissez-faire allocation exists and is unique. It is characterized by the following necessary and sufficient conditions:

\[
V^v(v^*, a^*) - d a^* \theta \sigma^2(v^*) = c 
\]  

(22)

\[
V^a(v^*, a^*) = d \theta \sigma^2(v^*) 
\]  

(23)

The laissez-faire level \(v^*\) of agro-biodiversity increases with increasing private risk; it is unaffected by increasing public risk; and the laissez-faire level \(a^*\) of financial insurance coverage is neither affected by an increase in private nor in public risk:

\[
\frac{dv^*}{d\theta} > 0 , \quad \frac{dv^*}{d\Theta} = 0 \quad \text{and} \quad \frac{da^*}{d\theta} = 0 , \quad \frac{da^*}{d\Theta} = 0 . 
\]  

(24)
**Proof:** see Appendix A.1.

Condition (22) states that the farmer will choose the level of agro-biodiversity so as to equate the marginal benefits and the marginal costs of agro-biodiversity. The marginal costs are given by the constant unit costs \( c \) on the right hand side. The marginal benefits are given by the expression on the left hand side and comprise two terms: the (private) insurance value of agro-biodiversity and the reduction in payments for financial insurance that results from the reduced variance of agro-ecosystem service due to a marginal increase in agro-biodiversity.

Likewise, Condition (23) states that the level of financial insurance coverage is chosen so as to equate the marginal benefits and the marginal costs of financial insurance, where the marginal benefit is the insurance value and the marginal costs are the (marginal) transaction costs.

As different forms of insurance, natural insurance from agro-biodiversity and financial insurance are substitutes: as financial insurance becomes more expensive, i.e. \( \delta \) increases, the farmer reduces his demand for financial insurance coverage and increases his level of agro-biodiversity. Put the other way: as financial insurance becomes cheaper, it drives out agro-biodiversity as the natural insurance. In any case, with financial insurance available, the farmer will choose a level of agro-biodiversity which is below the one that he would choose if financial insurance was not available.\(^8\)

An increase in the private risk, \( \theta \), leads the farmer to choose a higher level of agro-biodiversity, \( v^* \), as this provides him with increased natural insurance (Result 24). It does not lead the farmer, however, to choose a higher level of financial insurance coverage. The reason is that in the model of financial insurance considered here (cf. Section 2.2) the actuarially fair insurance premium is based on the extent of risk, so that with increasing risk the premium is also increasing. This increase in the real costs of insurance exactly counter-balances the increased need for financial insurance coverage, so that a change in the private risk, \( \theta \), does overall not have any impact on the demand for financial insurance, \( a^* \).

An increase in public risk, \( \Theta \), has no effect on a farmer’s private decision, as it purely affects the external benefits, and not the private benefits, of on-farm agro-biodiversity.

\(^8\)This level can be determined from setting \( a = 0 \) in Problem (21) and maximizing over \( v \). It is strictly smaller than \( v^* \) for all \( \delta < \rho \) and equals \( v^* \) for \( \delta \geq \rho \), i.e. in cases where financial insurance is so expensive that an optimizing farmer would not buy it. See Appendix A.1 for details.
3.3 Efficient allocation

The efficient allocation \((\hat{v}, \hat{a})\) is derived by choosing the level of agro-biodiversity \(v\) and financial insurance coverage \(a\) such as to maximize total welfare (Equation 12), subject to Constraints (6), (7), (9) and (10):

\[
\max_{v,a} W = \mu + \Upsilon - cv - \delta a \theta \sigma^2(v) - \frac{\rho}{2} (1 - a)^2 \theta \sigma^2(v) - \frac{\Omega}{2} \Theta \Sigma^2(v) .
\]  

(25)

The efficient allocation has the following properties.

**Proposition 2**

An (interior) solution to problem (25) exists and is unique. It is characterized by the following necessary and sufficient conditions:

\[
V^v(\hat{v}, \hat{a}) + V^{pub}(\hat{v}) - \delta \hat{a} \theta \sigma^2(\hat{v}) = c
\]  

(26)

\[
V^a(\hat{v}, \hat{a}) = \delta \theta \sigma^2(\hat{v})
\]  

(27)

The efficient level \(\hat{v}\) of agro-biodiversity increases with both increasing private and increasing public risk, and the efficient level \(\hat{a}\) of financial insurance coverage is neither affected by an increase in private nor in public risk:

\[
\frac{d\hat{v}}{d\theta} > 0 , \quad \frac{d\hat{v}}{d\Theta} > 0 \quad \text{and} \quad \frac{d\hat{a}}{d\theta} = 0 , \quad \frac{d\hat{a}}{d\Theta} = 0 .
\]  

(28)

**Proof:** see Appendix A.2

The properties of the efficient allocation are similar in structure to those of the laissez-faire allocation (cf. Proposition 1). The difference between the efficient and the laissez-faire allocation is that in the efficient allocation the positive externality that on-farm agro-biodiversity has on society at large in terms of a reduced variance of public benefits is fully captured: first order condition (26), which demands equality of marginal benefits and costs of agro-biodiversity, includes not only the private insurance value but also the public insurance value of agro-biodiversity.

Accordingly, the efficient level of agro-biodiversity increases not only with an increase in private risk, \(\theta\), but also with an increase in public risk, \(\Theta\).

3.4 Welfare effects of increasing environmental risks

Comparing the laissez-faire allocation (cf. Proposition 1) with the efficient allocation (cf. Proposition 2), it becomes apparent that there is market failure: Due to the external benefit of on-farm agro-biodiversity, the laissez-faire allocation is not efficient. In the laissez-faire allocation a private farmer chooses a level of agro-biodiversity that is too low compared to the socially optimal level because he does
not take into account the positive externality on society at large. As a result, welfare is lower in the laissez-faire allocation than in the efficient allocation.

**Proposition 3**
The laissez-faire level of agro-biodiversity is lower than the efficient level, while the level of financial insurance coverage is the same in both allocations. As a result, laissez-faire welfare is lower than welfare in the efficient allocation.

\[
\begin{align*}
    v^* &< \hat{v} , \\
    a^* &= \hat{a} , \\
    W^* &< \hat{W} .
\end{align*}
\]

**Proof:** see Appendix A.3

### 3.4.1 Ex-ante Pigouvian subsidy

In order to implement the efficient allocation, a regulator could impose a Pigouvian subsidy on agro-biodiversity. Denoting by \( \tau \) the subsidy per unit of actually employed agro-biodiversity \( v \), which is set prior to the resolution of uncertainty in the provision of the private and public ecosystem service (hence: *ex-ante* subsidy), the optimization problem of a private farmer under such regulation reads

\[
\max_{v,a} U = \mu - cv - \delta a \theta \sigma^2(v) - \frac{\rho}{2} (1 - a)^2 \theta \sigma^2(v) + \tau v .
\]

(32)

Comparing the first order conditions for the efficient allocation (Problem 25) and for the regulated allocation (Problem 32), we obtain the optimal subsidy rate \( \hat{\tau} \).

**Proposition 4**
The efficient allocation is implemented if a subsidy rate \( \hat{\tau} \) on agro-biodiversity is set with

\[
\hat{\tau} = -\frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) > 0 .
\]

(33)

The optimal subsidy rate increases with increasing public risk, \( \Theta \), and decreases with increasing private risk, \( \theta \):

\[
\frac{d\hat{\tau}}{d\Theta} > 0 \quad \text{and} \quad \frac{d\hat{\tau}}{d\theta} < 0 .
\]

(34)

**Proof:** see Appendix A.4.

The Pigouvian subsidy rate \( \hat{\tau} \) captures the positive externality of on-farm agro-biodiversity on society at large. It is exactly given by agro-biodiversity’s public insurance value (Equation 20). Hence, the optimal subsidy rate is higher, the higher the public insurance benefits of agro-biodiversity are.
The optimal subsidy rate $\tau$ can be interpreted as a measure of the extent of regulation necessary to internalize the externality, i.e. to solve the public-good problem. Thus, it can also be interpreted as a measure of the size of the externality. The size of the externality depends on the extent of private and public risk, $\theta$ and $\Theta$, because the level of agro-biodiversity depends on the risk faced by the farmer and, hence, on the level of natural insurance by agro-biodiversity he chooses.

Increasing risks have a clear-cut effect on the size of the externality. Condition (34) states that the optimal subsidy rate – that is, the size of the externality – decreases with increasing private risk, $\theta$. The intuitive reason for this result is that the farmer uses natural insurance to a greater extent the larger his private risk is. For that purpose he provides more on-farm agro-biodiversity thus also providing more of the public good. As a consequence, the externality decreases. Increasing public risk, $\Theta$, has two effects: as a direct effect, the public insurance value of a given level of agro-biodiversity increases (see Section 3.1). This effect increases the size of the externality that exactly equals the public insurance value (Equation 33). As an indirect effect, the efficient level of on-farm agro-biodiversity increases. Similar to the case of increasing private risk, this effect reduces the size of the externality. Proposition 4 shows that the direct effect unambiguously dominates the indirect effect of increasing public risk. Hence, the size of the externality increases with public risk (Result 34).

### 3.4.2 Ex-post compensation

The Pigouvian subsidy on agro-biodiversity derived in Proposition 4 is a payment that does not involve any uncertainty for the farmer. Such a policy may be called an *ex-ante* policy, as on-farm agro-biodiversity is subsidized for its *ex-ante* expected external benefit independently of the actual (uncertain) outcome. As an alternative policy instrument we consider a payment to the farmer in proportion to the actually occurring external benefit of on-farm agro-biodiversity after uncertainty is resolved. Such a scheme may be regarded as an *ex-post* compensation policy, as the farmer is paid *after* uncertainty is resolved.

To directly pay the farmer for the public ecosystem services the agro-ecosystem provides has been frequently proposed as a policy instrument under conditions of certainty (e.g. Hanley and Oglethorpe 1999). Here, we investigate how such a scheme works under conditions of uncertainty. The farmer would receive a payment $\beta B(v)$ in proportion to the actually realised external benefit $B(v)$ derived from the public ecosystem service after uncertainty is resolved, where $\beta$ is some positive number. The farmer would receive less than the public benefit, if $\beta < 1$, more if $\beta > 1$
and exactly the public benefit if $\beta = 1$. Note that the payment to the farmer is uncertain, as the external benefit of the public ecosystem service is uncertain, with $\mathcal{E}B(v) = \Upsilon$ and $\text{var} B(v) = \Theta \Sigma^2(v)$ (Equations 9 and 10). Under this policy, the farmer’s optimization problem is

$$\max_{v,a} \mu - cv - \delta a \theta \sigma^2(v) - \frac{\rho}{2} (1 - a)^2 \theta \sigma^2(v) + \beta \left( \Upsilon - \frac{\rho}{2} \Theta \Sigma^2(v) \right).$$

(35)

Comparing the first order conditions for the efficient allocation (Problem 25) and for the farmer’s optimal decision on agro-biodiversity (Problem 35), we obtain the following result

**Proposition 5**

The efficient allocation is implemented if and only if the farmer receives a payment of $\hat{\beta} B(v)$ for the public ecosystem service after uncertainty is resolved, with

$$\hat{\beta} = \Omega / \rho.$$  

(36)

**Proof:** see Appendix A.5.

According to Proposition 5 the farmer should receive a payment of $\hat{\beta} B(v) = \Omega / \rho B(v)$ that is smaller than the external benefit $B(v)$ from the ecosystem service if he is more risk-averse than society at large ($\rho > \Omega$). The reason is that, if payment to the farmer was at the full level of the external benefit he would supply more agro-biodiversity, involving higher costs of agro-biodiversity, than socially optimal. Only if the farmer’s individual degree of risk aversion equals society’s degree of risk aversion, $\rho = \Omega$, the farmer should be paid the full external benefit of on-farm agro-biodiversity. That is, only if the farmer and society at large are equally risk averse the same result is obtained under uncertainty as under conditions of certainty.

Under the optimal ex-post compensation scheme the farmer enjoys an additional marginal expected utility of agro-biodiversity equal to

$$\frac{d}{dv} \left[ \mathcal{E} [\hat{\beta} B(v)] - \frac{\rho}{2} \text{var} [\hat{\beta} B(v)] \right] = - \frac{\Omega}{2} \Theta \Sigma^2'(v).$$

(37)

Hence, the additional marginal expected utility due to the optimal ex-post policy (Equation 37) is exactly equal to the optimal subsidy rate $\hat{\tau}$ under the ex-ante policy (Equation 33). In this sense, the ex-ante policy and the ex-post policy are equivalent and both lead to the first-best allocation.

**3.4.3 Laissez-faire welfare**

After having studied the effect of increasing risks on the size of the externality, we now turn to the question of how increasing risks influence welfare. In a first-best...
world, where the externality is perfectly internalized, e.g. by the ex-ante Pigouvian subsidy (33) or the ex-post compensation (36), the answer to this question is simple: higher levels of both private and public risk are always welfare decreasing when both farmers and society at large are risk-averse.\footnote{This follows from applying the envelope theorem on welfare (Equation 12) with respect to \( \theta \) and \( \Theta \).}

This is not necessarily the case in the second-best world of the laissez-faire allocation where the externality of on-farm agro-biodiversity is present. Welfare in the laissez-faire allocation is given by (Equation 12 with 6, 7, 9 and 10)

\[
W^* \equiv \mu + \Upsilon - c v^* - \delta a^* \theta \sigma^2(v^*) - \frac{P}{2} (1 - a^*)^2 \theta \sigma^2(v^*) - \frac{\Omega}{2} \Theta \Sigma^2(v^*).
\]

We can immediately determine the impact of increasing public risk on laissez-faire welfare: Since society is risk-averse, increasing public risk decreases welfare. Since increasing public risk has no effect on the laissez-faire allocation (Proposition 1), there is no indirect effect that could reduce or even reverse this negative effect. Hence, welfare in the laissez-faire allocation unambiguously decreases with increasing public risk:

\[
\frac{dW^*}{d\Theta} < 0.
\]

Whether laissez-faire welfare increases or decreases with private risk, \( \theta \), depends on the relative size of two effects: (i) the direct effect of increased private risk is always negative (this is the only effect present in the first best); (ii) the indirect effect that increased private risk leads to an increased level of agro-biodiversity is positive (Proposition 1). The condition for whether one or the other effect dominates is given in the following proposition.

**Proposition 6**

With increasing private risk welfare in the laissez-faire allocation decreases / is unchanged / increases, i.e. \( dW^*/d\theta \leq 0 \), if and only if

\[
-\frac{\Omega}{2} \Theta \Sigma^2(v^*) \frac{dv^*}{d\theta} \leq \delta a^* \sigma^2(v^*) + \frac{P}{2} (1 - a^*)^2 \sigma^2(v^*) .
\]

**Proof:** see Appendix A.6.

The right-hand side of Condition (40) expresses the direct effect of increasing private risk: the higher private risk, the higher are the costs of financial insurance (the first term on the right hand side of 40) and the higher is the risk-premium of income from crop yield (the second term on the right hand side of 40). This direct effect decreases welfare. The left-hand side of Condition (40) captures the indirect...
effect of increasing private risk that on-farm biodiversity increases in the laissez-faire equilibrium (Proposition 1). This indirect effect leads to improved welfare, as the size of the externality is decreased (Proposition 4). The overall welfare effect depends on the balance between these two effects. In particular, if the indirect effect is sufficiently large welfare in the laissez-faire even increases with increasing private risk.

Using the conditions for the laissez-faire equilibrium (Proposition 1), Condition (40) can be expressed in the fundamental parameters of the model, and in terms of the public insurance value of agro-biodiversity (see Appendix A.6). We obtain the following alternative formulation of Proposition 6.

**Proposition 6’**

With increasing private risk welfare in the laissez-faire allocation decreases / is unchanged / increases, i.e. \( \frac{dW^*}{d\theta} \leq 0 \), if and only if

\[
V_{pub}^\tau(v^*) \leq c \left( \frac{\sigma^2(v^*) \sigma''(v^*)}{[\sigma''(v^*)]^2} \right). \tag{41}
\]

The left hand side of Condition (41) is the public (marginal) benefit, i.e. the public insurance value, of agro-biodiversity. Other things equal, with a larger public insurance value laissez-faire welfare is more likely to increase with private risk.

The first factor on the right-hand side of Condition (41) are the marginal costs of agro-biodiversity. Other things equal, laissez-faire welfare is more likely to increase with private risk the lower the marginal costs of agro-biodiversity are.

The second factor on the right-hand side of Condition (41) expresses the agro-ecosystem’s natural insurance function. In the example of an agro-ecosystem with isoeelastic natural insurance function (Equation 2) this factor becomes

\[
\frac{\sigma^2(v^*) \sigma''(v^*)}{[\sigma''(v^*)]^2} = \frac{\eta}{\eta - 1}. \tag{42}
\]

As \( \eta \) increases from 1 to infinity, this factor decreases from infinity to 1. Hence, the larger the agro-ecosystem’s natural insurance capacity, the smaller is this factor. Given the public insurance value of biodiversity and the costs of agro-biodiversity, a larger agro-ecosystem’s natural insurance capacity increases the likelihood that laissez-faire welfare increases with increasing private risk.

To summarize, Condition (41) states that laissez-faire welfare \( W^* \) decreases with private risk \( \theta \) if the agro-ecosystem is characterized by a low public insurance value, high marginal costs of agro-biodiversity and a low natural insurance capacity of the agro-ecosystem. Under these circumstances, the negative direct effect of private
risk to private farmers dominates over its positive indirect effect of increased agro-
biodiversity. So, an increase in private risk decreases total welfare. Interestingly, 
the reverse may also happen in the second-best world where the agro-biodiversity 
externality is not internalized: an increase in private risk may increase total welfare. 
This holds for an the agro-ecosystem and economic conditions that are characterized 
by a high natural insurance capacity, low costs and a high public insurance value 
of agro-biodiversity. Under these circumstances, the positive indirect effect, i.e. an 
increase in the level of agro-biodiversity and in the associated public and private 
insurance value, outweighs the negative direct effect of increased private risk.

4 Conclusions

We have studied how a risk-averse farmer manages his portfolio of agro-biodiversity 
so as to hedge his income risk. Our analysis captures two stylized facts: (i) On-
farm agro-biodiversity provides benefits not just at the farm level, but also provides 
external benefits. (ii) The variance of private and public benefits decreases with the 
level of agro-biodiversity. Thus, agro-biodiversity has both a private and a public 
natural insurance value.

Increasing environmental risks lead to a higher level of on-farm agro-biodiversity, 
because farmers use biodiversity’s natural insurance function to a greater extent. 
Yet, due to the external benefits of on-farm agro-biodiversity, the laissez-faire al-
location is not efficient. In order to study how this market failure is affected by 
increasing environmental risks we have analyzed how (i) the extent of regulation 
necessary to implement the efficient allocation and (ii) welfare in the laissez-faire 
allocation depend on the risk associated with the private and the public ecosystem 
service.

We found that the ex-ante Pigouvian subsidy, as a measure of the extent of 
efficient regulation in a first-best world, unambiguously decreases with the risk as-
associated with the private ecosystem service (crop yield), and increases with the risk 
associated with the public ecosystem service. Likewise, an ex-post compensation 
payment can cure this market failure problem. We have shown that, if the indi-
vidual farmer is more risk-averse than society at large, the compensation payment 
should be smaller than under certainty.

We also found that in a second-best world where such regulation does not exist, 
or is not properly enforced, it is even possible that increased private risk increases 
welfare. While this is, in principle, well-known from second-best theory, we have 
derived a specific condition on agro-ecosystem functioning under which this hap-
pens: increased private risk will have a positive impact on total welfare if the agro-ecosystem is characterized by a high natural insurance capacity, the marginal costs of agro-biodiversity are low, and its public insurance value is high.

These results are very relevant for agricultural and environmental policy. First, the socially optimal management of increasing environmental risks requires the optimal internalization of the environmental externality associated with agro-biodiversity. Existence of actuarially fair insurance against environmental risk is not sufficient for that sake. Second, the optimal policy response crucially depends on whether it is the private or the public environmental risk that is increasing. If the public environmental risk is increasing environmental policy (i.e. the ex-ante subsidy on on-farm agro-biodiversity) needs to be reinforced, and if the private environmental risk is increasing environmental policy needs to be relaxed. Third, insofar as direct compensation payments are used to stipulate farmers to provide uncertain public environmental benefits, and individual farmers are more risk-averse than society at large, compensation payments should be lower than the payments derived under the assumption of certainty. Fourth, if an optimal environmental regulation is not in place, welfare may be increasing or decreasing with increasing private risk. Yet, even in the case of increasing welfare, this is welfare-inferior to optimal environmental regulation. So, our result that laissez-faire welfare may be increasing due to increasing environmental risks should not be taken as an excuse for policy inaction.

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References


Appendix

A.1 Proof of Proposition 1

Written down explicitly, the first order conditions (22) and (23) for the interior solution of problem (21), which are obtained as \( \partial U / \partial v = 0 \) and \( \partial U / \partial a = 0 \), are

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \theta \sigma^2(v^*) = c \quad (A.1)
\]

\[
\rho (1 - a^*) \theta \sigma^2(v^*) = \delta \theta \sigma^2(v^*) \quad (A.2)
\]

Condition (A.2) can be solved to

\[
a^* = 1 - \frac{\delta}{\rho} \quad (A.3)
\]

Differentiating (A.1) with respect to \( \rho \) and using (A.3) yields

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \theta \sigma^2(v^*) \frac{dv^*}{d\rho} = \frac{1}{2} (1 - a^*)^2 \theta \sigma^2(v^*) \quad (A.4)
\]

\[
\frac{dv^*}{d\rho} = - \frac{\delta}{\rho} \frac{1}{2 \rho - \delta} \frac{\sigma^2(v^*)}{\sigma^2(v^*)} > 0 \quad (A.5)
\]

Differentiating (A.1) with respect to \( \delta \) and using (A.3) yields

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \theta \sigma^2(v^*) \frac{dv^*}{d\delta} = a^* \theta \sigma^2(v^*) \quad (A.6)
\]

\[
\frac{dv^*}{d\delta} = - \frac{a^*}{\frac{\rho}{2} (1 - a^*)^2 + \delta a^* \sigma^2(v^*)} \quad (A.7)
\]

\[
\frac{dv^*}{d\delta} = - \frac{1}{\delta} \frac{\rho - \delta}{\delta - \frac{\rho}{2}} \frac{\sigma^2(v^*)}{\sigma^2(v^*)} > 0 \quad (A.8)
\]

Differentiating (A.1) with respect to \( \theta \) and using (A.3) yields

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \theta \sigma^2(v^*) \frac{dv^*}{d\theta} = \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \sigma^2(v^*) \quad (A.9)
\]

\[
\frac{dv^*}{d\theta} = - \frac{1}{\theta} \frac{\sigma^2(v^*)}{\sigma^2(v^*)} > 0 \quad (A.10)
\]

Differentiating (A.1) with respect to \( \Theta \) and using (A.3) immediately yields

\[
\frac{dv^*}{d\Theta} = 0 \quad (A.11)
\]

Differentiating (A.3) with respect to \( \rho \) and \( \delta \) is straightforward and yields expressions for \( da^*/d\rho \) and \( da^*/d\delta \). As \( a^* \), according to Condition (A.3), does not depend on \( \theta \) or \( \Theta \) one has \( da^*/d\theta = da^*/d\Theta = 0 \).
A.2 Proof of Proposition 2

Written down explicitly, the first order conditions (26) and (27) for the interior solution of problem (25), which are obtained as \( \partial W/\partial v = 0 \) and \( \partial W/\partial a = 0 \), are

\[
- \left[ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right] \theta \sigma^2(\hat{v}) - \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) = c \quad (A.12)
\]

\[
\rho (1 - \hat{a}) \theta \sigma^2(\hat{v}) = \delta \theta \sigma^2(\hat{v}) \quad (A.13)
\]

Condition (A.13) can be solved to

\[
\hat{a} = 1 - \frac{\delta}{\rho} \quad (A.14)
\]

Differentiating (A.12) with respect to \( \rho \) and using (A.14) yields

\[
- \left\{ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right\} \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) \frac{d\hat{v}}{d\rho} = \frac{1}{2} (1 - \hat{a})^2 \theta \sigma^2(\hat{v}) \quad (A.15)
\]

\[
\frac{d\hat{v}}{d\rho} = \frac{-\frac{1}{2} \frac{\delta^2}{\rho^2} \theta \sigma^2(\hat{v})}{\delta \left( 1 - \frac{\delta}{2\rho} \right) \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v})} > 0 \quad (A.16)
\]

Differentiating (A.12) with respect to \( \Omega \) and using (A.14) yields

\[
- \left\{ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right\} \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) \frac{d\hat{v}}{d\Omega} = \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) \quad (A.17)
\]

\[
\frac{d\hat{v}}{d\Omega} = \frac{-\frac{1}{2} \Theta \Sigma^2(\hat{v})}{\delta \left( 1 - \frac{\delta}{2\rho} \right) \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v})} > 0 \quad (A.18)
\]

Differentiating (A.12) with respect to \( \delta \) and using (A.14) yields

\[
- \left\{ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right\} \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) \frac{d\hat{v}}{d\delta} = \hat{a} \theta \sigma^2(\hat{v}) \quad (A.19)
\]

\[
\frac{d\hat{v}}{d\delta} = \frac{-\left( 1 - \frac{\delta}{\rho} \right) \theta \sigma^2(\hat{v})}{\delta \left( 1 - \frac{\delta}{2\rho} \right) \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v})} > 0 \quad (A.20)
\]

Differentiating (A.12) with respect to \( \theta \) and using (A.14) yields

\[
- \left\{ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right\} \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) \frac{d\hat{v}}{d\theta} = \left[ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right] \sigma^2(\hat{v}) \quad (A.21)
\]

\[
\frac{d\hat{v}}{d\theta} = - \left[ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right] \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) > 0 \quad (A.22)
\]
Differentiating (A.12) with respect to \( \Theta \) and using (A.14) yields

\[
- \left\{ \frac{\rho}{2} \left( (1 - \hat{a})^2 + \delta \hat{a} \right) \left( \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) \right) \right\} \frac{d\hat{v}}{d\Theta} = \frac{\Omega}{2} \Sigma^2(\hat{v}) \tag{A.23}
\]

\[
\frac{d\hat{v}}{d\Theta} = -\frac{\Omega}{2} \Sigma^2(\hat{v}) \left( \left( \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right) \theta \sigma^2(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) \right) \geq 0 \tag{A.24}
\]

Differentiating (A.14) with respect to \( \rho \) and \( \delta \) is straightforward and yields expressions for \( d\hat{a}/d\rho \) and \( d\hat{a}/d\delta \). As \( \hat{a} \), according to Condition (A.14), does not depend on \( \Omega \), \( \theta \) or \( \Theta \) one has \( d\hat{a}/d\Omega = d\hat{a}/d\theta = d\hat{a}/d\Theta = 0 \).

A.3 Proof of Proposition 3

(i) From Conditions (A.3) and (A.14) it is apparent that \( a^* = \hat{a} \).

(ii) As \( a^* = \hat{a} \), Conditions (22) and (26) can be interpreted as equations of functions of the single variable \( v \) that determine the levels of \( v^* \) and \( \hat{v} \), respectively. Both conditions have as their right-hand side the constant \( c \), and as their left-hand side a strictly decreasing function of \( v \), so that \( v^* \) and \( \hat{v} \) are uniquely determined. As the term \( V_{pub}(v) = -\frac{\Omega}{2} \Theta \Sigma^2(v) \) is strictly positive for all \( v \), the left-hand side of Condition (26) is strictly greater than the left-hand side of Condition (22) for all \( v \). As a result the value of \( v \) that equates the left-hand side with the right-hand side is strictly greater for Condition (26) than for Condition (22), i.e. \( \hat{v} > v^* \).

(iii) \( \hat{W} \geq W^* \) by definition of the efficient allocation as the allocation that maximizes \( W \). Strict inequality follows from strict concavity of \( W \) in \( \hat{v} \) and \( \hat{v} > v^* \).

A.4 Proof of Proposition 4

The first order conditions for the interior solution of Problem (32), which are obtained as \( \partial U/\partial v = 0 \) and \( \partial U/\partial a = 0 \), are

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \theta \sigma^2(v^*) + \tau = c \tag{A.25}
\]

\[
a^* = 1 - \frac{\delta}{\rho} \tag{A.26}
\]

Comparison of Condition (A.25) with Condition (A.12) reveals that

\[
v^* = \hat{v} \quad \text{for} \quad \tau = \hat{\tau} = -\frac{\Omega}{2} \Theta \Sigma^2(\hat{v}) \tag{A.27}
\]
Employing results (A.16), (A.18), (A.20), (A.22) and (A.24), the comparative statics of $\hat{\tau}$ are

\[
\frac{d\hat{\tau}}{d\Omega} = -\frac{1}{2} \Theta \Sigma'^2(\hat{v}) - \frac{\Omega}{2} \Theta \Sigma''(\hat{v}) \frac{d\hat{v}}{d\Omega} \\
= -\frac{1}{2} \Theta \Sigma'^2(\hat{v}) \left\{ 1 - \frac{\Omega}{2} \Theta \Sigma''(\hat{v}) \left[ 1 - \frac{\delta}{\delta a} \theta \Sigma''(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma''(\hat{v}) \right] \right\} > 0 \quad \text{(A.28)}
\]

\[
\frac{d\hat{\tau}}{d\rho} = -\frac{\Omega}{2} \Theta \Sigma''(\hat{v}) \frac{d\hat{v}}{d\rho} < 0 \quad \text{(A.29)}
\]

\[
\frac{d\hat{\tau}}{d\delta} = -\frac{\Omega}{2} \Theta \Sigma''(\hat{v}) \frac{d\hat{v}}{d\delta} < 0 \quad \text{(A.30)}
\]

\[
\frac{d\hat{\tau}}{d\theta} = -\frac{\Omega}{2} \Sigma'^2(\hat{v}) - \frac{\Omega}{2} \Theta \Sigma''(\hat{v}) \frac{d\hat{v}}{d\theta} < 0 \quad \text{(A.31)}
\]

\[
\frac{d\hat{\tau}}{d\Theta} = -\frac{\Omega}{2} \Sigma'^2(\hat{v}) \left\{ 1 - \frac{\Omega}{2} \Theta \Sigma''(\hat{v}) \left[ \frac{\rho}{2} (1 - \hat{a})^2 + \delta a \right] \theta \Sigma''(\hat{v}) + \frac{\Omega}{2} \Theta \Sigma''(\hat{v}) \right\} > 0 \quad \text{(A.32)}
\]

### A.5 Proof of Proposition 5

The first order conditions for the interior solution of Problem (35), which are obtained as $\partial U/\partial v = 0$ and $\partial U/\partial a = 0$, are

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \theta \Sigma''(v^*) - \beta \frac{\rho}{2} \Theta \Sigma''(v^*) = c \quad \text{(A.33)}
\]

\[
a^* = 1 - \frac{\delta}{\rho} \quad \text{(A.34)}
\]

Comparison of Condition (A.33) with Condition (A.12) reveals that

\[
v^* = \hat{v} \quad \text{for} \quad \beta = \Omega/\rho. \quad \text{(A.35)}
\]

### A.6 Proof of Proposition 6

Differentiating $W^*$ (Equation 38) with respect to $\theta$ yields

\[
\frac{dW^*}{d\theta} = - \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \theta \Sigma''(v^*) - \frac{\Omega}{2} \Theta \Sigma''(v^*) \frac{dv^*}{d\theta} \quad \text{(A.36)}
\]

So,

\[
\frac{dW^*}{d\theta} < 0 \quad \Leftrightarrow \quad -\frac{\Omega}{2} \Theta \Sigma''(v^*) \frac{dv^*}{d\theta} < \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \theta \Sigma''(v^*) \quad \text{(A.37)}
\]

Employing (A.9), this condition can be expressed explicitly as

\[
-\frac{\Omega}{2} \Theta \Sigma''(v^*) \frac{dv^*}{d\theta} < \left( \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right) \theta \frac{\Sigma''(v^*) \Sigma''''(v^*)}{\Sigma''(v^*)} \frac{dv^*}{d\theta} \quad \text{(A.38)}
\]
Using (16) and (20), this leads to

\[ V_{\text{pub}}(v^\ast) \leq \left( V^v(v^\ast, a^\ast) - \delta a^\ast \theta \sigma^2(v^\ast) \right) \frac{\sigma^2(v^\ast) \sigma^{2''}(v^\ast)}{[\sigma^{2'}(v^\ast)]^2}. \]  

(A.39)

Using (A.1) yields Condition (41).


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