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RESEARCH REPORTS HAM – PI

THE RELATION OF CHILDREN'S PERFORMANCES IN SPATIAL TASKS AT TWO DIFFERENT SCALES OF SPACE

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This study investigates the relation between performances of fourth-graders in spatial tasks with depictive material in the classroom and orientation tasks in real space. The children completed a paper and pencil test and a map-based orientation test on campus. A correlational analysis revealed that the children's performances in small-scale spatial tasks are related to their performances in large-scale spatial tasks. Moreover, classes of small-scale tasks that require mental transformations concerning the self and concerning objects are related to large-scale tasks that involve the update of the self-to-landmark relations in real space and the map-environment relation, respectively. Both classes contributed to the prediction of performances in map-based orientation tasks that require a constant update of map-self-landmark relations.

INTRODUCTION

Solving spatial tasks is recommended in geometry classes in primary school since doing so helps children to "grasping space", i.e. it contributes to a child's thoughtful interaction with the three-dimensional space in which they live, play and move (Freudenthal, 1973). The demands on spatial tasks in geometry education are therefore twofold: on the one hand, they should foster a child's ability to interact successfully with space. On the other hand, spatial tasks should allow a child to integrate and enrich individual spatial experiences while solving them. In order to accomplish both goals, spatial tasks should ideally be introduced into geometry classes in both abstract-depictive spatial settings in the classroom *and* in concrete-navigational spatial settings in real space (OECD, 2004, p.36).

Current studies in mathematics education emphasize the importance of spatial tasks in both contexts but typically investigate those in settings that include only written or small material (e.g. Logan et al., 2017). Researcher may do so because they assume that the performances in spatial tasks in depictive settings equal the performances in navigational settings in real space. However, empirical evidence on whether and to which extent performances in both contexts are related has never been provided. This study addresses that gap at a conceptual and empirical level.

THEORETICAL BACKGROUND

Cognitive psychologists conceptualize spatial tasks with depictive material, such as paper and pencil tests, as *small-scale spatial tasks*, since they rely on a stimulus that can be perceived from one single vantage point. They conceptually contrast them to

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large-scale spatial tasks that require the locomotion of the subject towards multiple viewing points in order to be completed and may require the successful interpretation of a spatial representation such as a map (Montello, 1993, Hegarty et al., 2006).

In order to comply spatial tasks at both scales of space, children typically need to understand the interplay between different spatial positions in space and the visual appearance of object configurations. Hereby, the child needs to be able to encode and mentally manipulate three changing relations between the different objects, the self, and the environment: *object-to-environment relations*, *self-to-object relations* and *self-to-environment relations* (e.g. Hegarty et al., 2006).

Small-scale spatial tasks can be differentiated according to two classes of mental transformations demands that are necessary in order to solve them: (1) tasks that require *object-based transformations*, i.e. tasks that require the mental movement of a set of objects in the environment (OB*)*, and (2) tasks that require *egocentric perspective transformations*, i.e. tasks that require the mental movement of the own point of view in relation to a set of objects *(EGO)*. Both classes have been found to be distinct not only on the conceptual level, but also on an empirical level (e.g. Kozhevnikov et al., 2006).

Large-scale spatial tasks can also be conceptualized in a differentiated way according to different task demands. The memorizing of landmarks (important recognizable "objects") without providing maps has been studied under the perspective of individual differences in the performance to keep track of changing self-to-landmark relations in the environment that enable the formation of a *cognitive map* (e.g. Hegarty et al., 2006). *Static map use*, that focuses on aligning a map with the environment in order to draw directional inferences from it while not moving in space has been studied with respect to individual differences in the performance of recognizing and correcting misaligned relations between the map and the environment (e.g. Shepard & Hurwitz, 1984). Finally, *dynamic map use*, that requires the subject to keep track of the self-location and orientation on the map while moving in space or to navigate to landmarks, has been investigated with respect to individual differences in the performance to update self-to-map, self-to-environment, and map-to-environment relations (e.g. Liben et al., 2008). Although it has been highlighted that large-scale spatial tasks need to be conceptualized in a differentiated way (e.g. Kozhevnikov et al., 2006), the distinction of the classes outlined above is less studied from an empirical point of view.

Divergent results have been reported concerning the relation between performances of children in small- and large-scale spatial tasks. Those have been shown to be either totally dissociated (Quaiser-Pohl et al., 2004) or partially related (Liben et al., 2013). The latter study as well as similar studies with adults (Liben et al., 2008, Kozhevnikov et al., 2006) highlighted the potential role of single OB and EGO tasks as common and unique predictors of diverse large-scale spatial tasks.

PURPOSE OF THE STUDY

The goal of this study was to investigate the relation between the performances in small-scale spatial tasks and the performances in large-scale spatial tasks of primary school children. We aimed to examine whether classes of paper and pencil tasks were reliable and unique predictors of different of map-based orientation tasks. Moreover, we intended to assess whether patterns of unique prediction where generalizable for classes of map-based orientation tasks.

METHOD

Participants and stimuli

240 (111 m, 129 f) fourth graders from the north of Germany participated in the study on the campus of our university. The children were aged between 9 and 12 years (m=10.29, SD=.48). Each child completed a paper and pencil test in a group and a map-based orientation test in large-scale space individually at the same day with a break of at least 20min for cognitive recover.

Paper and Pencil Test

The Paper and Pencil Test consisted of eight small-scale spatial tasks, four of them measuring performances in tasks that require egocentric (EGO) transformations and four of them measuring performances in tasks that require object-based (OB) transformations. We developed EGO tasks mostly from the scratch and designed tasks that require the children to relate field views of various object configurations to the corresponding positions in plan views. One task was an adoption of the Guilford-Zimmermann-Boat test for children. The OB tasks consisted of an adoption of Ekstrom's Card Rotation Test, an adoption of the Vandenberg Mental Rotation Test, and adoption of the Paper Folding Test for children. We further designed a task that requires the children to imagine going along a path on a map and decide on each crossing whether they turned left or right.

We tested the quality of our tasks in a pilot study with N=222 children, making sure that our self-developed test has acceptable psychometrical characteristics and is construct-valid (EGO tasks are empirically separable from OB tasks).

Map-based orientation test

The map-based test consisted of eight tasks with three items each that were integrated in a treasure hunt on the campus (Table 1). One task was performed at the starting location in the beginning (*Rot*) and two at the end (*MDisk, MFlag)* of the treasure hunt. For all other tasks, we subsequently led the children to three flags (*Dots, Dir, HP, Read*) and finally encouraged them to place the disks on the campus (*Disks*). During the whole test, the children were not allowed to turn their map.

The test consisted of tasks that operationalized cognitive mapping processes (CM), the performance of mentally aligning a map in space in order to draw inferences from it while being static (MapUse) and the performance of keeping the orientation on where they are on a map while moving in space (MapOrtn). Those tasks represented the underlying construct in the large-scale test.

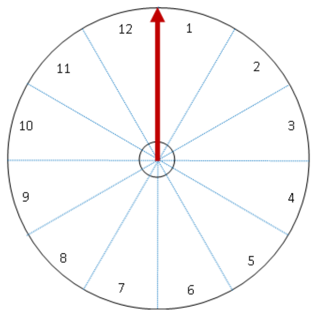
Task	Description	Measure
MFlags /MDisks (CM)	Requires the child to point to the locations of the flags/disks without using a map.	Correctness of the directions was measured with the help of an arrow and circle device that served as help for indicating directions.
Rot (MapUse)	Requires the child to indicate directions of landmarks on the map while taking different canonical viewing directions in the real space.	
Read (MapUse)	Requires the child to indicate directions of landmarks on the map while standing next to a flag.	
HP (MapUse)	Requires the child to point to the starting point.	
Dir (MapOrtn)	Requires the child to indicate the viewing direction once arrived at a flag.	
Dots (MapOrtn)	Requires the child to indicate the location of the current position with a coloured sticker on the map after walking from flag to flag.	Deviations of the stickers.
Disks (MapOrtn)	Requires the child to put a disk in the environment according to the location marked in the map.	Deviation of the disk.

Table 1: Large-scale spatial tasks in the map-based orientation test.

Data treatment

We encoded our data and analysed patterns of missing values in the map-based orientation test. We ensured that missing values are at least MAR and applied multiple imputations before further analysis. We computed 30 multiple imputations according to Si & Reiter's method (2013) using the R package *NPBayesImpute*, computed sum scores for all tasks and finally pooled the data sets using the R package *semtools*, which allowed us to extract one single empirical correlation matrix.

RESULTS

To investigate the relation of classes of small-scale tasks with the map-based orientation tasks, we computed the factor scores for EGO and OB. Both factor scores correlated with $r=.64$ ($p<0.001$). The result demonstrates that they indeed share a considerable amount of variance that will be considered further in our correlation analysis.

Relations between classes of small-scale tasks and single large-scale tasks

In a first step, we computed the pairwise correlations between the factor scores of the two classes of small-scale tasks and the set of large-scale tasks. As shown in Table 2, both classes correlate significantly with performances in the large-scale tasks. Only the performances in pointing towards the memorized locations of the flags did not correlate with either of two classes of small-scale tasks.

	MDisk	MFlag	Disks	Dir	Dots	Read	HP	Rot
EGO	.24**	.12	.41**	.28**	.43**	.38**	.20**	.20**
OB	.17**	.10	.42**	.29**	.45**	.40**	.23**	.26**
Residual EGO-OB	.18**	.07	.19**	.11	.19**	.16*	.06	.04
Residual OB-EGO	.01	.02	.21**	.15*	.22**	.21**	.14*	.18**

* two-tailed p< 0.05 ** two-tailed p<0.01

Table 2: Correlations and semipartial correlations for the task wise analysis.

To examine whether performances in EGO or in OB tasks predicted unique variance in the large-scale measures, we computed semipartial correlations (see also Table 2). After partialling out the shared variance between performances in EGO and OB tasks, for some large-scale tasks, only one of the two classes of small-scale task became significant, indicating that they predicted unique variance in the respective task. For instance, only the semipartial correlation between EGO and the performances in memorizing the locations of the disks became significant. Thus, cognitive resources that are unique to EGO tasks – performing egocentric transformations – appeared to have affected performances in the task *MDisk*, which requires updating of self-to-environment relations.

In two other cases, only the semipartial correlation with OB became significant. For those tasks, cognitive resources that are unique to OB tasks – performing object-based transformations while keeping the self-to-environment relation constant – affected the performances. OB tasks predicted therefore unique variance in two tasks that required the correct alignment of a map in space (*HP* and *Rot*). In the case of the task *Dir*, the semipartial correlation of EGO was also almost significant (p=0.07). For this reason, we did not interpret OB tasks to be unique sources of variance in this task. For the tasks *Disks*, *Dots*, and perhaps *Dir*, for both classes of small-scale tasks the semipartial correlations became significant. Thus, processing resources that are unique to OB tasks and unique to EGO tasks appeared to have affected the performance in those tasks that require keeping oriented after moving in space.

Relations between classes of small-scale tasks and classes of large-scale tasks

To further analyze the initial results at the broader level of classes of large-scale tasks, we performed a CFA using *R lavaan* in order to show that our tasks loaded on the factors that we derived from the literature. For the sake of shortness, we do not present the full analysis here. For each of the 30 data sets, the fit indices revealed a CFI>.99, a TLI>.98, RMSEA<0.05, and a non-significant chi-squared test showed that the model did not derive essentially from the data (see Hu & Bentler, 1999). We conjectured that the tasks in our map-based orientation test are clustered in accordance with the classes that we conceptualized from the literature. We then computed the corresponding factor scores and calculated correlations.

	Factor *CM:* Cognitive Mapping	Factor *MapUse:* Static Map Use	Factor 3 *MapOrtn:* Dynamic Map Use
EGO	.22**	.35**	.49**
OB	.15*	.41**	.50**
Residual EGO-OB	.15*	.11	.21**
Residual OB-EGO	.02	.25**	.25**

* two-tailed p< 0.05 ** two-tailed p<0.01

Table 4: Correlations and semipartial correlations between the performances in EGO and OB and the three classes of map-based orientation tasks.

As we show in Table 4, the correlation between EGO and OB with the class of cognitive mapping tasks was significant (p = 0.001 and p=0.02, respectively). In order to determine whether one of them predicted unique variance in tasks of cognitive mapping, we computed semipartial correlations. Once the shared variance of OB and EGO was partialled out, only the semipartial correlation between EGO and the first factor of large-scale tasks was significant (p=0.01), whereas the semipartial correlation between OB and the factor was not. Cognitive resources that are unique to EGO tasks, in particular egocentric mental transformations appear to have affected the performances in this self-to-environment representation factor. Similarly, correlations between EGO and OB with the class of static map use tasks, were highly significant (p<0.001). Once the shared variance between EGO and OB was partialled out, the semipartial correlation between OB and the second class was still significant (p<0.001), whereas the semipartial correlation to EGO was not (p=0.09). Thus, cognitive resources that are unique for object-based transformations, in particular the correct mental update of relations between objects and the environment, appear to have affected the performances in this map alignment factor. Finally, an analysis of the correlations between EGO and OB with the class of dynamic map use tasks revealed highly significant correlations (p<0.001). Even after partialling out the shared variance, both EGO and OB were still significant predictors when it came to dynamic mapping (p=0.001 and p<0.001, respectively). Thus, cognitive resources that are unique to EGO and to OB tasks, egocentric and object-based transformations, appear to have both affected the performances in this map-based orientation factor.

In summary, our empirical findings provide evidence that the children's performances in small-scale spatial tasks are related to the performances in large-scale spatial tasks.

The two classes of small-scale spatial tasks, EGO and OB both predicted the performance of large-scale spatial tasks at the level of single tasks and classes of them. After partialling out shared variance between EGO and OB tasks, however, we identified EGO tasks to be the only reliable predictor of cognitive mapping tasks and OB the only reliable predictor of static map use tasks. However, both classes are reliable predictors of dynamic map use tasks.

DISCUSSION

The results described above support the idea that spatial tasks should be used in a differentiated way in mathematics education. Our findings provide evidence that the performances of small-scale tasks are partially, but not fully related to performances in large-scale tasks. One possible explanation might be related to the underlying spatial abilities that enable solving those tasks with a certain performance. They probably rely on common cognitive processes that allow for the processing of small- and large-scale information such as the encoding of the spatial information and the representation in working memory (cf. Hegarty et al., 2006). Investigating these processes might be an important next step in mathematics education research. Our findings highlight, that large-scale tasks should be conceptualized in a differentiated way. Furthermore, the patterns of correlation reported within this study suggest a taxonomic classification of large-scale tasks that is analogous to one classification of small-scale spatial tasks. Indeed, tasks that demand egocentric mental transformations in small-scale space find their analogue on tasks that rely on a correct update of the self-to-landmark and self-to-environment relations, which can be interpreted as egocentric transformations in large-scale space (e.g. Kozhevnikov et al., 2006). Tasks that demand object-based transformations in small-scale space find their analogue in tasks that rely on updating processes between the map and the environment that can be interpreted as object-based transformations in large-scale space (e.g. Shepard & Hurwitz, 1984). Finally, dynamic map use tasks seem to be determined by a subsequent composition of egocentric transformations that allow to update self-to-map and self-to-landmark relations in the environment, and object-based transformations that allow to mentally updates the relation between the map and the environment while moving. This finding is in line with previous suggestions that dynamic map use requires two sets of mental transformations (Aretz & Wickens, 1992).

In future research, the relation between performances in small- and large-scale spatial tasks could be investigated not only at the level of classes of small-scale tasks, but also at the level of single tasks. This could point towards a set of good spatial tasks for practices in classroom and beyond. Furthermore, the relation could be studied at the latent level of the assumed underlying spatial abilities as well. Shifting the empirical investigations from the manifest to the latent level would result in an explicit modelling of measurement errors that probably allows for computing measurement error-free correlations.

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References

Aretz, A. J., & Wickens, C. D. (1992). The mental rotation of map displays. *Human performance*, *5*(4), 303–328.

Freudenthal, H. (1973). The case of geometry. In *Mathematics as an educational task* (pp. 401–511). Springer, Dordrecht.

Hegarty, M., Montello, D. R., Richardson, A. E., Ishikawa, T., & Lovelace, K. (2006). Spatial abilities at different scales: Individual differences in aptitude-test performance and spatial-layout learning. *Intelligence*, *34*(2), 151–176.

Hu, L. T., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural equation modeling: a multidisciplinary journal*, *6*(1), 1–55.

Kozhevnikov, M., Motes, M. A., Rasch, B., & Blajenkova, O. (2006). Perspective-taking vs. mental rotation transformations and how they predict spatial navigation performance. *Applied Cognitive Psychology*, *20*(3), 397–417.

Liben, L.S., Myers, L.J., & Kastens, K.A. (2008). Locating oneself on a map in relation to person qualities and map characteristics. In C. Freska, N.S. Newcombe, P. Gärdenfors, & S. Wölfl (Eds.), *Spatial cognition VI*: Learning, reasoning, and talking about space (pp. 171–187). Heidelberg, Germany: Springer-Verlag.

Liben, L. S., Myers, L. J., Christensen, A. E., & Bower, C. A. (2013). Environmental-Scale Map Use in Middle Childhood: Links to Spatial Skills, Strategies, and Gender. *Child development*, *84*(6), 2047–2063.

Logan, T., Lowrie, T. & Ramful, A. (2017). Decoding map items through spatial orientation: performance differences across grade and gender. In B. Kaur, W.K. Ho, T.L. Toh & B.H. Choy (Eds.), *Proc. 41st Conf. of the Int. Group for the Psychology of Mathematics Education* (vol. 3, pp. 193-200). Singapore: PME.

Organisation for Economic Co-operation and Development. (2004). *The PISA 2003 assessment framework: mathematics, reading, science and problem solving knowledge and skills*. OECD Publishing.

Quaiser-Pohl, C., Lehmann, W., & Eid, M. (2004). The relationship between spatial abilities and representations of large-scale space in children—a structural equation modeling analysis. *Personality and Individual Differences*, *36*(1), 95–107.

Shepard, R. N., & Hurwitz, S. (1984). Upward direction, mental rotation, and discrimination of left and right turns in maps. *Cognition*, *18*(1), 161–193.

Si, Y., & Reiter, J. P. (2013). Nonparametric Bayesian multiple imputation for incomplete categorical variables in large-scale assessment surveys. *Journal of Educational and Behavioral Statistics*, *38*(5), 499–521.