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Agro-biodiversity as natural insurance and the development of financial insurance markets

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Abstract. Agro-biodiversity can provide natural insurance to risk averse farmers. We employ a conceptual ecological-economic model to analyze the choice of agro-biodiversity by risk averse farmers who have access to financial insurance. We study the implications for individually and socially optimal agro-ecosystem management and policy design when on-farm agro-biodiversity, through ecosystem processes at higher hierarchical levels, generates a positive externality on other farmers. We show that for the individual farmer natural insurance from agro-biodiversity and financial insurance are substitutes. While an improved access to financial insurance leads to lower agro-biodiversity, the effects on the market failure problem (due to the external benefits of on-farm agro-biodiversity) and on welfare are determined by properties of the agro-ecosystem and agro-biodiversity’s external benefits. We derive a specific condition on agro-ecosystem functioning under which, if financial insurance becomes more accessible, welfare in the absence of regulation increases or decreases.

JEL-Classification: Q1, Q57, H23, D62

Keywords: agro-biodiversity, ecosystem services, agro-ecosystem management, insurance, risk-aversion, uncertainty

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1 Introduction

Farmers traditionally grow a variety of crops in order to decrease the adverse impact of uncertain environmental and market conditions. That is, they use agrobiodiversity as a form of natural income insurance. In this paper, we study how risk-averse farmers manage their portfolio of agro-biodiversity to hedge their income risk from uncertain environmental conditions, and how this management decision is being affected by the availability of financial insurance. Obviously, the two options – natural insurance through agro-biodiversity and financial insurance from the market – are substitutes for risk-averse farmers (Baumgärtner 2007). So, the price of financial insurance has an impact on the level of agro-biodiversity cultivated on the farm for risk-management purposes: as financial insurance becomes cheaper, it drives out agro-biodiversity as a form of natural insurance.

In the trade-off between financial insurance and natural insurance through agro-biodiversity, a market failure problem arises from the fact that agro-biodiversity does not only provide private on-farm benefits, but also gives rise to public benefits such as improved pollination or control of pests or diseases, i.e. reduced income risk, on neighboring farms. As a general result, the privately determined level of on-farm agro-biodiversity is lower than the socially optimal one (Heal et al. 2004). In particular, such market failure stems from the risk-changing characteristics of agro-biodiversity and risk-averse behavior of private farmers (Baumgärtner 2007, Quaas and Baumgärtner, in press). In this paper, we study whether this risk-related market failure in the allocation of agro-biodiversity is worsened or lessened by improved access to financial insurance.

Agro-biodiversity’s private and public insurance function, and its interrelation with financial insurance from the market, has different economic dimensions. Our analysis therefore builds upon, and combines, different strands in the economic literature.

Agro-biodiversity as a form of natural insurance

A number of studies have analyzed the contribution of crop diversity to the mean and variance of agricultural yields (Smale et al. 1998, Schläpfer et al. 2002, Widawsky and Rozelle 1998, Zhu et al. 2000) and to the mean and variance of farm income (Di Falco and Perrings 2003, 2005, Di Falco et al. 2005). One result is that agrobiodiversity may increase the mean level, and decrease the variance, of crop yields. This result is perfectly in line with evidence that emerged from recent theoretical, experimental and observational research in ecology about the role of biodiversity for the provision of ecosystem services (Hooper et al. 2005, Kinzig et al. 2002, Loreau et
It has been conjectured that risk averse farmers use crop diversity in order to hedge their income risk (Birol et al. 2005a, 2005b, Di Falco and Perrings 2003). Since agro-biodiversity has an insurance value for farmers, they tend to employ a higher level of agro-biodiversity in the face of uncertainty (Baumgartner 2007, Quaas and Baumgartner, in press). The extent to which farmers rely on agro-diversity as a natural insurance may be affected by agricultural policies such as subsidized crop yield insurance or direct financial assistance (Di Falco and Perrings, 2005). In this respect, agro-biodiversity plays a similar role for risk averse farmers as other risk changing production factors, such as e.g. nitrogen fertilizer or pesticides (Horowitz and Lichtenberg 1993, 1994a, 1994b).

**Interaction of natural and financial insurance**

Instead of making use of natural insurance, farmers can also buy financial insurance to hedge their income risk. For example, in the USA for over one hundred years crop yield insurance is offered to manage agricultural risk. Since traditional crop yield insurance is particularly vulnerable to classical insurance problems such as moral hazard or adverse selection (e.g. Luo et al. 1994), considerable effort is recently spent to develop alternative possibilities of financial insurance for farmers, e.g. index-based insurance contracts (Miranda and Vedenov 2001, Skees et al. 2002, World Bank 2004).

While this effort to develop instruments of financial insurance is motivated by the idea that reducing income risk is beneficial for farmers, some studies have shown that financial insurance tends to have ecologically negative effects. Horowitz and Lichtenberg (1993, 1994a, 1994b) show that financially insured farmers are likely to undertake riskier production – with higher nitrogen and pesticide use – than uninsured farmers do. A similar result is pointed out by Mahul (2001), assuming a weather-based insurance. Wu (1999) empirically estimates the impact of insurance on the crop mix and its negative results on soil erosion in Nebraska (USA).

The underlying economic reason is that agro-biodiversity as a form of natural insurance and financial insurance from the market are substitutes, so that improved access to the latter drives out the former (Baumgartner 2007, Quaas and Baumgartner 2006). In the insurance economics literature, the analysis of the trade-off between ‘self insurance’ (by acting such as to reduce a potential income loss) or ‘self protection’ (by acting such as to reduce the probability of an income loss) on the one hand, and ‘market insurance’ on the other hand goes back to Ehrlich and Becker (1972). One standard result is that self insurance and market insurance are substitutes, with the result that market insurance, as it becomes cheaper, may drive out self insurance.
Underprovision / Overuse of public good

Since agro-biodiversity has not only a private insurance function but provides public insurance benefits as well, there is a potential public good problem associated with the private provision of agro-biodiversity (Heal et al. 2004). For example, the extent of genetic diversity in food crops is important as it affects the risk of attack by pathogens. A drop in diversity increases this risk. Farmers may not take this into account when making crop choices, leading to what from a social perspective is an inadequate level of agro-diversity.

The conventional wisdom on the use (or provision) of a public good under uncertainty seems to be that the more uncertainty and the higher the risk aversion of individual decision makers, the less severe is the problem of overuse (or under-provision) of the public good (Bramoullé and Treich 2005, Sandler and Sterbenz 1990, Sandler et al. 1987). In a sense, this literature suggests that private uncertainty and risk-aversion increase the efficiency of the private provision of public goods. The focus in this literature is on the properties of the utility function, while the production of the public good (or public bad) is typically modelled in a trivial way, i.e. one unit of money spent on providing the public good equals one unit of the public good provided.

Quaas and Baumgartner (in press) have shown that in realistic settings, in which the production of a public good – such as a public insurance function – is generated in a complex system – such as a multi-scale ecosystem – things become ambiguous. They find that ecosystem management and environmental policy depend on the extent of uncertainty and risk-aversion as follows: (i) Individual effort to increase the level of biodiversity unambiguously increases. However, the free-rider problem may decrease or increase, depending on the characteristics of the ecosystem and its management; in particular, (ii) the size of the externality may decrease or increase, depending on how individual and aggregate management effort influence biodiversity; and (iii) the welfare loss due to free-riding may decrease or increase, depending on how biodiversity influences ecosystem service provision.

If agro-biodiversity has not only a private but also a public insurance value, the interrelationship between natural and financial insurance becomes more complex, too. Quaas and Baumgartner (in press) have shown that while improved access to financial insurance leads to a lower level of agro-biodiversity, the effect on the public-good problem and on overall welfare is ambiguous and determined by agro-ecosystem properties.

In this paper, we bring together the various ideas about agro-biodiversity and fi-
nancial insurance, and analyze them in a unified formal framework. We analyze how a risk-averse farmer makes use of the natural insurance function of agro-biodiversity and of financial insurance. In particular, we study the question of how availability of financial insurance affects the underprovision of agro-biodiversity and social welfare when on-farm agro-biodiversity generate both a private benefit and, via ecological processes at higher hierarchical levels, also public benefits.

The analysis is based on a conceptual ecological-economic model. Crop yield is random because of exogenous sources of risk (e.g. weather, diseases or pests); its statistical distribution (mean and variance) is determined by the level of agro-biodiversity. The level of on-farm agro-biodiversity not only determines the distribution of farm income, but also generates external benefits. The farmer is risk-averse and chooses the level of agro-biodiversity so as to maximize the expected utility of farm income. When making this choice, he has also access to financial income insurance.

We show that natural insurance through agro-biodiversity and financial insurance are substitutes. Hence, availability of financial insurance reduces the demand for natural insurance through agro-biodiversity and, thus, leads to a reduction in agro-biodiversity. In particular, the lower the costs of financial insurance are (i.e. the more actuarially fair the risk premium of financial insurance is), the lower is the resulting level of agro-biodiversity. Yet, the effects of an improved access to financial insurance on the market failure problem (due to the external benefits of on-farm agro-biodiversity) and on welfare are ambiguous. We derive a specific condition on agro-ecosystem functioning under which, if financial insurance becomes more accessible, welfare in the absence of regulation increases or decreases.

These results are highly policy relevant. While at first sight the introduction of, or improved access to, financial and insurance markets seems to be beneficial to farmers from a welfare point of view, our results demonstrate that – depending on agro-ecosystem properties – it may have adverse welfare effects.

The paper is organized as follows. In Section 2, we specify the ecological-economic model. The analysis and results are presented in Section 3, with all proofs and formal derivations contained in the Appendix. Section 4 discusses the results and concludes.

2 Ecological-economic model

We consider a farmer who manages an agro-ecosystem for the service, i.e. crop yield, it provides. Due to stochastic fluctuations in environmental conditions the provision
of the agro-ecosystem service is uncertain. Its statistical distribution depends on
the state of the agro-ecosystem in terms of agro-biodiversity, which is determined
by the farmer’s management decision. As a result, the statistical distribution of
agro-ecosystem service and, hence, of income depend on ecosystem management.
We capture these relationships in a stylized ecological-economic model as follows.

2.1 Agro-ecosystem management

The farmer chooses a level $v$ of agro-biodiversity, say by selecting a portfolio of
different crop varieties. Given the level of agro-biodiversity $v$, the agro-ecosystem
provides the farmer with the desired service, i.e. total crop yield, at a level $s$ which is
random. For simplicity we assume that the agro-ecosystem service directly translates
into monetary income and that its mean level $\mathcal{E} s = \mu$ is independent of the level of
agro-biodiversity and constant.\footnote{Empirical evidence suggests that $\mu$ may depend on $v$ (see Section 1). We explored the impact of such relationships in previous versions of the model. Here, we neglect such a dependence of $\mu$ on $v$ as it complicates the analysis while not adding further insights into the insurance dimension of the issue under study.} The variance of agro-ecosystem service depends on
the level of agro-biodiversity $v$ as follows

$$\text{var } s = \sigma^2(v) \quad \text{where} \quad \sigma^2'(v) < 0 \text{ and } \sigma^2''(v) \geq 0 . \quad (1)$$

For illustrative purpose, we will consider the following specific example:

$$\sigma^2(v) = \sigma_0 v^{1-\eta} \quad \text{with} \quad \eta > 1 . \quad (2)$$

The constant $\eta$ parameterizes the natural insurance function of the agro-ecosystem: the larger $\eta$, the stronger does the variance of agro-ecosystem service (total crop
yield) decline with the level of agro-biodiversity.

2.2 Financial insurance

In order to analyze the influence of availability of financial insurance on the farmers’
choice of agro-biodiversity, we introduce financial insurance in a simple and stylized
way. We assume that the farmer has the option of buying financial insurance under
the following contract: (i) The farmer chooses the fraction $a \in [0, 1]$ of insurance
coverage. (ii) He receives (pays)

$$a (\mathcal{E} s - s) \quad (3)$$

\footnote{For a formal motivation in terms of agro-biodiversity’s insurance value, see Section 3.1}
from (to) the insurance company as an actuarially fair indemnification benefit (insur-
ance premium) if his realized income is below (above) the mean income.\textsuperscript{3} In order
to abstract from any problems related to informational asymmetry, we assume that
the statistical distribution as well as the actual level \(s\) of agro-ecosystem service
are observable to both insurant and insurance company. (iii) In addition to (3),
the farmer pays the transaction costs of insurance. The costs of insurance over and
above the actuarially fair insurance premium, which are a measure of the ‘real’ costs
of insurance to the farmer, are assumed to follow the cost function
\[
\delta a \text{ var } s ,
\]
where the parameter \(\delta \geq 0\) describes how actuarially unfair is the insurance contract.
The costs increase linearly with the insured part of income variance. This captures
in the simplest way the idea that the costs of insurance increase with the ‘extent’
of insurance. Throughout the analysis we assume \(\delta < \rho\) to exclude corner solutions
where a change in \(\delta\) would have no effect on the farmer’s behavior.

The main focus of our analysis will lie in the comparative statics with respect
to the parameter \(\delta\). Thereby we interpret a decrease in \(\delta\) as an improvement in the
access to, or reduction of the costs of, financial insurance.\textsuperscript{4}

\section*{2.3 Farmer’s income, preferences and decision}

The farmer chooses the level of agro-biodiversity \(v\) and financial insurance coverage
\(a\). A higher level of agro-biodiversity carries costs \(c > 0\) per unit of agro-biodiversity.
These costs may be due to increased cropping, harvesting and marketing effort, and
are purely private. Adding up income components, the farmer’s (random) income \(y\)
is given by
\[
y = (1 - a) s - c v + a \mathcal{E} s - \delta a \text{ var } s .
\]
Since the agro-ecosystem service \(s\) is a random variable, net income \(y\) is a random
variable, too. The uncertain part of income is captured by the first term in
Equation (5), while the other components are certain. Obviously, increasing \(a\) to
one allows the farmer to reduce the uncertain income component down to zero.

\textsuperscript{3}This benefit/premium-scheme is actuarially fair, because the insurance company has an ex-
pected net payment stream of \(\mathbb{E}[a(\mathcal{E}s - s)] = 0\). This model of insurance is fully equivalent to
the traditional model of insurance (e.g. Ehrlich and Becker 11: 627) where losses compared with
the maximum income are insured against and the insurant pays a constant insurance premium
irrespective of actual income. In this traditional model, the net payment would exactly amount to
(3); for a formal proof see Quaas and Baumgärtner (in press, Appendix A.1).

\textsuperscript{4}The parameter \(\delta\) could be treated as a policy variable, as it could be influenced by subsidies
or taxes. Yet, in this paper we treat \(\delta\) as an exogenous parameter.
The mean $\mathcal{E}y$ and the variance $\text{var } y$ of the farmer’s income $y$ are determined by the mean and variance of agro-ecosystem service, which depends on the level of agro-biodiversity (Equation 1),

$$\mathcal{E}y = \mu - c v - \delta a \sigma^2(v) \quad \text{and} \quad \text{var } y = (1 - a)^2 \sigma^2(v),$$

Mean income is given by the mean level of agro-ecosystem service $\mu$, minus the costs of agro-biodiversity $c v$ and the costs of financial insurance $\delta a \sigma^2(v)$. For an actuarially fair financial insurance contract ($\delta = 0$), mean income equals mean net income from agro-ecosystem use, $\mu - c v$. The variance of income vanishes for full financial insurance coverage, $a = 1$, and equals the full variance of agro-ecosystem service, $\sigma^2(v)$, without any financial insurance coverage, $a = 0$.

The farmer is assumed to be non-satiated and risk-averse with respect to his uncertain income $y$. There exists empirical evidence on how agro-biodiversity influences the mean and variance of agro-ecosystem services, but hardly on the full statistical distribution. This restricts the class of risk preferences which can meaningfully be represented in our ecological-economic model to utility functions which depend only on the first and second moment of the probability distribution, i.e. on the mean and the variance. Specifically, we assume the following expected utility function, where $\rho > 0$ is a parameter describing the farmer’s degree of risk aversion (Arrow 1965, Pratt 1964):

$$U = \mathcal{E}y - \frac{\rho}{2} \text{var } y.$$  

### 2.4 External benefits of agro-biodiversity

The farmer’s private decision on the level of agro-biodiversity $v$ affects not only his private income risk, as expressed by the variance of on-farm agro-ecosystem service, $\text{var } s$ (Equation 1), but also causes external effects. Assume that $B(v)$ captures the sum of external benefits of on-farm agro-biodiversity $v$, such as improved pollination or control of pests or diseases, i.e. reduced income risk, on neighboring farms. In particular, we shall assume that the external benefit of agro-biodiversity essentially consists in a reduction of public risk, i.e. in a reduction of the variance of some

\footnote{More general utility functions of the mean-variance type would complicate the analysis without generating further insights.}

\footnote{Quaas and Baumgärtner (in press) provide an explicit model of many farmers that shows how public benefits may arise from individual biodiversity management.}
public ecosystem service:

$$\mathcal{EB}(v) = \Upsilon$$  \hspace{1cm} (9)

$$\text{var } B(v) = \Sigma^2(v) \quad \text{where} \quad \Sigma^2(v) < 0 \text{ and } \Sigma^{2''}(v) \geq 0. \hspace{1cm} (10)$$

The external welfare effect of on-farm agro-biodiversity is

$$\mathcal{EB} - \Omega \frac{1}{2} \text{var } B,$$  \hspace{1cm} (11)

where $\Omega > 0$ is a parameter describing the degree of social risk aversion. Furthermore, we assume that the private and the public risks associated with $v$ are uncorrelated. The total (i.e. private plus external) welfare effect of on-farm agro-biodiversity, thus, is

$$W = \mathcal{E}y + \mathcal{EB} - \rho \frac{1}{2} \text{var } y - \Omega \frac{1}{2} \text{var } B. \hspace{1cm} (12)$$

3 Analysis and results

The analysis proceeds in four steps: First, we identify agro-biodiversity’s private and public insurance value (Section 3.1). Next, we discuss the laissez-faire allocation which arises if the farmer maximizes his expected private utility from farm income (Section 3.2). Then, we study the efficient allocation which is obtained by maximizing social welfare (Section 3.3). Finally, we investigate how policy measures to internalize the externalities and welfare are influenced by the access to financial insurance, as described by the parameter $\delta$ (Section 3.4).

3.1 The insurance value of agro-biodiversity

In order to precisely define the insurance value of agro-biodiversity, recall that by choosing the level of agro-biodiversity $v$ and the fraction of financial insurance coverage $a$ the farmer actually chooses a particular income lottery, which in our model is characterized by the mean $\mathcal{E}y = \mu - cv - \delta a \sigma^2(v)$ and variance $\text{var } y = (1-a)^2 \sigma^2(v)$ (Equations 6, 7). These are determined by $v$ and $a$ and, therefore, one may speak of ‘the lottery $(v, a)$’.

One standard method of valuing the riskiness of a lottery to a decision maker is to calculate the risk premium $R$ of the lottery, which is defined as the amount of money that leaves the decision maker equally well off, in terms of utility, between

\footnote{In case of coorrelated private and public risks Equation (12) would generalize to $W = \mathcal{E}y + \mathcal{EB} - \frac{1}{2} \text{var } y - \frac{1}{2} \text{var } B - \gamma \text{covar}(y, B)$.}
the two situations of (i) receiving for sure the expected pay-off from the lottery $E_y$ minus the risk premium $R$, and (ii) playing the risky lottery with random pay-off $y$ (e.g. Dasgupta and Heal 1979: 381, Kreps 1990: 84). With utility function (8), the risk premium $R$ of a lottery with mean pay-off $E_y$ and variance $\text{var } y$ is simply given by:

$$R = \frac{\rho}{2} \text{var } y .$$

In the model employed here the risk premium of the farmer’s income lottery thus depends on the level of agro-biodiversity $v$ and of financial insurance coverage $a$:

$$R(v, a) = \frac{\rho}{2} (1 - a)^2 \sigma^2(v) .$$

The insurance value of agro-biodiversity can now be defined based on the risk premium of the lottery $(v, a)$ (Baumgärtner 2007).

**Definition 1**

The insurance value $V^v$ of agro-biodiversity $v$ is given by the change of the risk premium $R$ of the lottery $(v, a)$ due to a marginal change in the level of agro-biodiversity $v$:

$$V^v(v, a) := -\frac{\partial R(v, a)}{\partial v} .$$

Thus, the insurance value of agro-biodiversity is the marginal value of agro-biodiversity in its function to reduce the risk premium of the farmer’s income risk from harvesting uncertain agro-ecosystem services. Being a marginal value, it depends on the existing level of agro-biodiversity $v$. It also depends on the actual level of financial insurance coverage $a$. The minus sign in the defining Equation (15) serves to express agro-biodiversity’s ability to reduce the risk premium of the lottery $(v, a)$ as a positive value. Applying Definition 1 to Equation (14), one obtains the following result for the insurance value of agro-biodiversity in this model.

**Proposition 1**

The insurance value $V^v(v, a)$ of biodiversity is given by

$$V^v(v, a) = -\frac{\rho}{2} (1 - a)^2 \sigma^2(v) > 0 .$$

From Equation (16) it is apparent that the insurance value of agro-biodiversity has an objective, a subjective and an institutional dimension. The objective dimension is captured by the sensitivity of the variance of agro-ecosystem services to changes in agro-biodiversity, $\sigma^2(v)$; the subjective dimension is captured by the farmer’s degree of risk aversion, $\rho$; and the institutional dimension is captured by the farmer’s extent of financial insurance coverage, $a$, which depends on institutional conditions (see below). The insurance value of agro-biodiversity $V^v$ increases with the sensitivity of the
variance of agro-ecosystem services to changes in agro-biodiversity, $|\sigma'^2|$, and with the degree $\rho$ of the farmer’s risk aversion. It decreases with the farmer’s extent of financial insurance coverage, $a$. In the extreme, for vanishing subjective risk-aversion, $\rho = 0$, or for full financial insurance coverage, $a = 1$, agro-biodiversity’s insurance value vanishes. As a function of the level $v$ of agro-biodiversity, the insurance value $V^v(v, a)$ decreases: as agro-biodiversity becomes more abundant (scarcer), its insurance value decreases (increases).

In the example of specification (2), agro-biodiversity’s insurance value $V^v(v, a)$ is isoelastic with respect to changes in the level of agro-biodiversity $v$, and $\eta$ expresses this elasticity.\footnote{Formally, $-v \frac{\partial V^v(v, a)}{\partial v}/V^v(v, a) \equiv \eta$.} That is, an increase of agro-biodiversity by 1% always leads to an increase of its insurance value by $\eta \%$. This motivates the interpretation of $\eta$ as the agro-ecosystem’s natural insurance function.

One can also define the insurance value of financial insurance as

$$V^a(v, a) := - \frac{\partial R(v, a)}{\partial a}. \quad (17)$$

With Expression (14) for the risk premium of the income lottery $(v, a)$, the insurance value $V^a(v, a)$ of financial insurance is thus given by

$$V^a(v, a) = \rho (1 - a) \sigma^2(v). \quad (18)$$

From Equation (18) it is apparent that the insurance value of financial insurance also has an objective, a subjective and an institutional dimension. The objective dimension is captured by the variance of agro-ecosystem services, $\sigma^2$, which represents the extent of potential environmental risk; the subjective dimension is captured by the farmer’s degree of risk aversion, $\rho$; and the institutional dimension is captured by the farmer’s extent of financial insurance coverage, $a$, which depends on institutional conditions (see below). The insurance value of financial insurance $V^a$ increases with the variance of agro-ecosystem services, $\sigma^2$, i.e. with environmental risk, and with the degree $\rho$ of the farmer’s risk aversion. It decreases with the farmer’s extent of actual financial insurance coverage, $a$. In the extreme, for vanishing subjective risk-aversion, $\rho = 0$, vanishing environmental risk, $\sigma^2 = 0$, or for full financial insurance coverage, $a = 1$, the value of financial insurance vanishes.

So far, we have been discussing agro-biodiversity’s private insurance value to an individual farmer, based on the private risk premium $R(v, a)$ (Equation 14) of the farmer’s private income lottery. Beyond that, agro-biodiversity also has a public insurance value. On-farm agrobiodiversity has an additional risk-reducing value due
to its external benefit (11), i.e. there exists a public risk premium, 

$$R^{pub}(v) = \frac{\Omega}{2} \text{var} B = \frac{\Omega}{2} \Sigma^2(v),$$

which is in addition to the private one, giving rise to a public insurance value of 

$$V^{pub}(v) = -\frac{\partial R^{pub}(v)}{\partial v} = -\frac{\Omega}{2} \Sigma^2'(v) > 0.$$ 

The total insurance value of on-farm agro-biodiversity then is the sum of the private and the public insurance value.

### 3.2 Laissez-faire allocation

As laissez-faire allocation \((v^*, a^*)\) we consider the allocation in which the farmer individually chooses the level of agro-biodiversity \(v\) and financial insurance coverage \(a\) so as to maximize his expected private utility (Equation 8) subject to constraints (6) and (7). Formally, the farmer’s decision problem is

$$\max_{v,a} U = \mu - c v - \delta a \sigma^2(v) - \frac{\rho}{2} (1 - a)^2 \sigma^2(v).$$

The laissez-faire allocation has the following properties.

**Proposition 2**

An (interior) laissez-faire allocation exists and is unique. It is characterized by the following necessary and sufficient conditions:

$$V^v(v^*, a^*) - \delta a^* \sigma^2'(v^*) = c$$

$$V^a(v^*, a^*) = \delta \sigma^2(v^*)$$

The laissez-faire levels of both agro-biodiversity and financial insurance coverage increase with the degree of risk-aversion:

$$\frac{dv^*}{d\rho} > 0 \quad \text{and} \quad \frac{da^*}{d\rho} > 0.$$ 

The laissez-faire level \(v^*\) of agro-biodiversity increases, and the laissez-faire level \(a^*\) of financial insurance coverage decreases, with the costs of financial insurance:

$$\frac{dv^*}{d\delta} > 0 \quad \text{and} \quad \frac{da^*}{d\delta} < 0.$$ 

**Proof:** see Appendix A.1.

Condition (22) states that the farmer will choose the level of agro-biodiversity so as to equate the marginal benefits and the marginal costs of agro-biodiversity. The
marginal costs are given by the constant unit costs $c$ on the right hand side. The marginal benefits are given by the expression on the left hand side and comprise two terms: the insurance value of agro-biodiversity and the reduction in payments for financial insurance that results from the reduced variance of agro-ecosystem service due to a marginal increase in agro-biodiversity.

Likewise, Condition (23) states that the level of financial insurance coverage is chosen so as to equate the marginal benefits and the marginal costs of financial insurance, where the marginal benefit is the insurance value and the marginal costs are the (marginal) transaction costs. This condition can be rearranged into

$$a^* = 1 - \frac{\delta}{\rho},$$

which states that the farmer will choose the level of financial insurance coverage as follows. In the absence of transaction costs, i.e. for $\delta = 0$, he chooses full coverage by financial insurance, i.e. $a^* = 1$. As transaction costs of financial insurance increase, i.e. for $\delta > 0$, he chooses partial coverage by financial insurance, $0 < a^* < 1$, and if transaction costs are so high that $\delta = \rho$ he chooses no financial insurance coverage, $a^* = 0$.9

Both the level of agro-biodiversity and the level of financial insurance coverage increase with the degree of the farmer’s risk-aversion (Result 24), since both instruments allow him to hedge his income risk. As different forms of insurance the two are substitutes: as financial insurance becomes more expensive, i.e. $\delta$ increases, the farmer reduces his demand for financial insurance coverage and increases his level of agro-biodiversity (Result 25). Put the other way: as financial insurance becomes cheaper, it drives out agro-biodiversity as the natural insurance. In any case, with financial insurance available, the farmer will choose a level of agro-biodiversity which is below the one that he would choose if financial insurance was not available.10

### 3.3 Efficient allocation

The efficient allocation $(\hat{v}, \hat{a})$ is derived by choosing the level of agro-biodiversity $v$ and financial insurance coverage $a$ so as to maximize total welfare (Equation 12), subject to Constraints (9), (10), (6) and (7):

$$\max_{v,a} W = \mu + \Upsilon - cv - \delta a \sigma^2(v) - \frac{\rho}{2} (1 - a)^2 \sigma^2(v) - \frac{\Omega}{2} \Sigma^2(v).$$

---

9Recall that we assume $\delta \leq \rho$ throughout in order to focus on interior solutions. For $\delta > \rho$, the optimization problem (21) would have a corner solution, $a^* = 0$, with $da^*/d\rho = da^*/d\delta = 0$.

10This level can be determined from setting $a = 0$ in Problem 21 and maximizing over $v$. It is strictly smaller than $v^*$ for all $\delta < \rho$ and equals $v^*$ for $\delta \geq \rho$, i.e. in cases where financial insurance is so expensive that an optimizing farmer would not buy it.
The efficient allocation has the following properties.

**Proposition 3**

An (interior) solution to problem (27) exists and is unique. It is characterized by the following necessary and sufficient conditions:

\[
V^v(\hat{v}, \hat{a}) + V^{pub}(\hat{v}) - \delta \hat{a} \sigma^2(\hat{v}) = c \tag{28}
\]

\[
V^a(\hat{v}, \hat{a}) = \delta \sigma^2(\hat{v}) \tag{29}
\]

The efficient levels of both agro-biodiversity and financial insurance coverage increase with the degree of individual risk-aversion:

\[
\frac{d\hat{v}}{d\rho} > 0 \quad \text{and} \quad \frac{d\hat{a}}{d\rho} > 0 . \tag{30}
\]

The efficient level of agro-biodiversity increases with, and the efficient level of financial insurance coverage is unaffected by, the degree of social risk-aversion:

\[
\frac{d\hat{v}}{d\Omega} > 0 \quad \text{and} \quad \frac{d\hat{a}}{d\Omega} = 0 . \tag{31}
\]

The efficient level \(\hat{v}\) of agro-biodiversity increases, and the efficient level \(\hat{a}\) of financial insurance coverage decreases, with the costs of financial insurance:

\[
\frac{d\hat{v}}{d\delta} > 0 \quad \text{and} \quad \frac{d\hat{a}}{d\delta} < 0 . \tag{32}
\]

**Proof:** see Appendix A.2

The properties of the efficient allocation are very similar in structure to those of the laissez-faire allocation (cf. Proposition 2). The difference between the efficient and the laissez-faire allocation is that in the efficient allocation the positive externality, which a private farmer’s effort has on society at large in terms of a reduced variance of public benefits, is fully internalized: first order condition (28), which demands equality of marginal benefits and costs of agro-biodiversity, includes not only the private insurance value but also the public insurance value, i.e. the full social insurance value, of agro-biodiversity.

This changes the effect that an increase in the transaction costs of financial insurance has on the management effort and financial insurance coverage in magnitude, but not in sign. Hence, the same arguments hold which support Proposition 2: with increasing transaction costs \(\delta\) of financial insurance it is optimal to substitute financial insurance by natural insurance.

As in the laissez-faire allocation, the efficient levels of agro-biodiversity, \(\hat{v}\), and financial insurance coverage, \(\hat{a}\), increase with the degree of individual risk aversion, \(\rho\).
3.4 Welfare effects of improved access to financial insurance

Comparing the laissez-faire allocation (cf. Proposition 2) with the efficient allocation (cf. Proposition 3), it becomes apparent that there is market failure: Due to the external benefit of on-farm agro-biodiversity, the laissez-faire allocation is not efficient. In the laissez-faire allocation a private farmer chooses a level of agro-biodiversity that is too low compared to the socially optimal level, because he does not take into account the positive externality on society at large. As a result, welfare is lower in the laissez-faire allocation than in the efficient allocation.

Proposition 4

The laissez-faire level of agro-biodiversity is lower than the efficient level, while the level of financial insurance coverage is the same in both allocations. As a result, laissez-faire welfare is lower than welfare in the efficient allocation.

\[ v^* < \hat{v}, \quad (33) \]
\[ a^* = \hat{a}, \quad (34) \]
\[ W^* < \hat{W}. \quad (35) \]

Proof: see Appendix A.3

In order to implement the efficient allocation, a regulator could impose a Pigouvian subsidy on agro-biodiversity. Denoting by \( \tau \) the subsidy per unit of \( v \), the optimization problem of a private farmer under such regulation then reads

\[
\max_{v,a} U = \mu - c v - \delta a \sigma^2(v) - \frac{\rho}{2} \left( 1 - a \right)^2 \sigma^2(v) + \tau v . \quad (36)
\]

Comparing the first order conditions for the efficient allocation (Problem 27) and for the regulated allocation (Problem 36), we obtain the optimal subsidy \( \hat{\tau} \).

Proposition 5

The efficient allocation is implemented if a subsidy \( \hat{\tau} \) on agro-biodiversity is set with

\[
\hat{\tau} = -\frac{\Omega}{2} \Sigma^2(\hat{v}) > 0 . \quad (37)
\]

The optimal subsidy increases with the degree \( \Omega \) of social risk aversion, and decreases with the degree \( \rho \) of individual risk aversion and with the costs \( \delta \) of financial insurance:

\[
\frac{d\hat{\tau}}{d\Omega} > 0 , \quad \frac{d\hat{\tau}}{d\rho} < 0 , \quad \frac{d\hat{\tau}}{d\delta} < 0 . \quad (38)
\]

Proof: see Appendix A.4.

The Pigouvian subsidy \( \hat{\tau} \) captures the positive externality of on-farm agro-biodiversity on society at large. It is exactly given by agro-biodiversity’s public insurance value.
(Equation 20). Hence, the optimal subsidy is higher, the higher the public insurance benefits of agro-biodiversity are.

The optimal subsidy \( \hat{\tau} \) can be interpreted as a measure of the extent of regulation necessary to internalize the externality, i.e. to solve the public-good problem. Thus, it can also be interpreted as a measure of the size of the externality.

Clearly, the size of the externality depends on the costs \( \delta \) of financial insurance. The effect of higher costs of financial insurance on the market failure is unambiguous. Condition (38) states that increasing costs of financial insurance decrease the market failure.

After having studied the effect of financial insurance on the size of the externality, we now turn to the question of how increased costs of financial insurance influence welfare. In a first-best economy, where the external effect is perfectly internalized, e.g. by the Pigouvian subsidy (37), the answer to this question is simple: higher costs of financial insurance are always welfare decreasing in a first-best world.\(^{11}\)

This is not necessarily the case in the second-best world of the laissez-faire allocation where the externality of on-farm agro-biodiversity is present. Whether welfare in the laissez-faire allocation (Equation 12)

\[
W^* \equiv \mu + \Upsilon - c v^* - \delta a^* \sigma^2(v^*) - \frac{\rho}{2} (1 - a^*)^2 \sigma^2(v^*) - \frac{\Omega}{2} \Sigma^2(v^*)
\]  

(39)

increases or decreases with the costs of financial insurance, \( \delta \), depends on the relative size of two effects: (i) the direct effect of increased insurance costs is always negative (this is the only effect present in the first best); (ii) the indirect effect that increased costs of financial insurance lead to an increased level of agro-biodiversity is positive (Proposition 2). The condition for whether one or the other effect dominates is given in the following proposition.

**Proposition 6**

With increasing costs of financial insurance welfare in the laissez-faire allocation decreases / is unchanged / increases, i.e. \( dW^*/d\delta \leqslant 0 \), if and only if

\[
-\frac{\Omega}{2} \Sigma^2(v^*) \frac{dv^*}{d\delta} \leqslant a^* \sigma^2(v^*) ,
\]  

(40)

which is equivalent to

\[
V^{pub}(v^*) \leqslant \left( V^v(v^*, a^*) - \delta a^* \sigma^2(v^*) \right) \frac{\sigma^2(v^*) \sigma^2''(v^*)}{\left[ \sigma^2(v^*) \right]^2}
\]  

(41)

---

\(^{11}\)This follows from applying the envelope theorem on total welfare (12) with respect to \( \delta \).
Proof: see Appendix A.5.

The right hand side of Condition (40) expresses the direct effect that expenditures for financial insurance increase with $\delta$. This effect decreases welfare. The left hand side of Condition (40) captures the indirect effect that on-farm biodiversity increases with $\delta$ (Proposition 2). Welfare is improved by the increase in $v^*$ weighted by a factor of $-\frac{\Omega}{2} \Sigma^2(v^*) > 0$ which quantifies the positive externality of the private choice of on-farm agro-biodiversity on society at large. The overall welfare effect depends on the balance between these two effects. In particular, if the indirect effect is sufficiently large welfare in the laissez-faire even increases with the costs of financial insurance.

Condition (40) can be expressed in the fundamental parameters of the model, and in terms of the private and public insurance value of agro-biodiversity (Condition 41). On the left hand side is the public (marginal) benefit, i.e. the public insurance value, of agro-biodiversity. On the right hand side is the private (marginal) benefit of agro-biodiversity, i.e. the private insurance value plus the indirect benefit of reduced costs of financial insurance, weighted by a factor of $\frac{\sigma^2(v^*) \sigma^2''(v^*)}{[\sigma^2'(v^*)]^2}$ which expresses the agro-ecosystem’s natural insurance function. In the example of an agro-ecosystem with isoelastic natural insurance function (Equation 2) this factor becomes

$$\frac{\sigma^2(v^*) \sigma^2''(v^*)}{[\sigma^2'(v^*)]^2} = \frac{\eta}{\eta - 1} = const. \quad (42)$$

As $\eta$ increases from 1 to infinity, this factor decreases from infinity to 1. So, the larger the agro-ecosystem’s natural insurance function, the smaller is this factor.

With this, Condition (41) states that laissez-faire welfare $W^*$ decreases with the costs $\delta$ of financial insurance if the agro-ecosystem is characterized by a low natural insurance function, the private insurance value of agro-biodiversity is high, and its public insurance value is low. Under these circumstances, the negative direct effect of financial insurance costs to private farmers dominates over its positive indirect effect of increased agro-biodiversity. So, an increase in private insurance costs decreases total welfare. Interestingly, the reverse may also happen in the second-best world where the agro-biodiversity externality is not internalized: an increase in private insurance costs may increase total welfare. This holds for a situation in which the agro-ecosystem is characterized by a high natural insurance function, the private insurance value of agro-biodiversity is low, and its public insurance value is high. Under these circumstances, the positive indirect effect, i.e. an increase in the level of agro-biodiversity and the associated public and private insurance value, outweighs the negative direct effect of increased costs of financial insurance.
After having studied the effect of improved access to financial insurance on laissez-faire welfare, we now look at how improved access affects the welfare loss from the market failure, which is due to the external benefits of agro-biodiversity. This welfare loss in the laissez-faire allocation compared with the efficient allocation is given by

$$\hat{W} - W^* = -c \hat{v} - \delta \hat{a} \sigma^2(\hat{v}) - \frac{\rho^2}{2} (1 - \hat{a})^2 \sigma^2(\hat{v}) - \frac{\Omega}{2} \Sigma^2(\hat{v})$$

$$- \left[-c v^* - \delta a^* \sigma^2(v^*) - \frac{\rho^2}{2} (1 - a^*)^2 \sigma^2(v^*) - \frac{\Omega}{2} \Sigma^2(v^*) \right], \quad (43)$$

where $$v^* < \hat{v}$$ and $$a^* = \hat{a}$$ so that $$\hat{W} - W^* > 0$$ (Proposition 4). The properties of the welfare loss are as follows:

**Proposition 7**

With increasing costs of financial insurance the welfare loss from market failure in the allocation of agro-biodiversity increases / decreases / is unchanged, i.e.

$$d(\hat{W} - W^*)/d\delta \geq 0, \text{ if and only if}$$

$$\frac{d}{d\delta} (\hat{W} - W^*) \geq 0$$

$$\Leftrightarrow V^{pub}(v^*) \leq \left(V^{v}(v^*, a^*) - \delta a^* \sigma^2(v^*) \right) \left(1 - \frac{\sigma^2(\hat{v})}{\sigma^2(v^*)} \right) \frac{\sigma^2(v^*) \sigma^{2''}(v^*)}{[\sigma^2(v^*)]^2} \quad (44)$$

**Proof:** see Appendix A.6.

Condition (44) about the welfare loss $$\hat{W} - W^*$$ is essentially the same as Condition (41) about the laissez-faire welfare level $$W^*$$, amended by a factor of $$1 - \sigma^2(\hat{v})/\sigma^2(v^*)$$, which may take on values between zero and one depending on the agro-ecosystem’s natural insurance function. So, essentially all interpretations of Proposition 6 carry over to the interpretation of Proposition 7. The additional factor of $$1 - \sigma^2(\hat{v})/\sigma^2(v^*)$$ in Condition (44) implies that the larger the deviation of the laissez-faire level of agro-biodiversity $$v^*$$ from its efficient level $$\hat{v}$$, the greater are the chances that the welfare loss increases with the costs of financial insurance.

### 4 Conclusions

We have analyzed how a risk-averse farmer manages his portfolio of agro-biodiversity so as to hedge his income risk. The ecological-economic model captures two stylized facts: (i) On-farm agro-biodiversity provides benefits not just at the farm level, but also provides external benefits. (ii) The variance of private and public benefits
decreases with the level of agro-biodiversity. Thus, agro-biodiversity has a natural insurance function.

Financial insurance is a substitute for natural insurance from agro-biodiversity. As a consequence, higher costs of financial insurance lead to a higher demand for natural insurance, and thus, to a higher level of agro-biodiversity. Put the other way around, introducing institutions for, or improving access to, financial insurance leads to a lower level of agro-biodiversity, as farmers substitute natural insurance from agro-biodiversity by financial insurance.

Due to the external benefits of on-farm agro-biodiversity, the laissez-faire allocation is not efficient. In order to study how this market failure is affected by the availability of financial insurance we have analyzed how (i) the extent of regulation necessary to implement the efficient allocation and (ii) how welfare in the laissez-faire allocation depend on the transaction costs of financial insurance.

We found that the Pigouvian subsidy, as a measure of the extent of efficient regulation in a first-best world, unambiguously decreases with the costs of financial insurance. We also found that in a second-best world where such regulation does not exist, or is not properly enforced, it is even possible that improved access to financial insurance decreases welfare. While this is, in principle, well-known from second-best theory, we have derived a specific condition on agro-ecosystem functioning under which this happens (Conditions 41 and 44): improved access to financial insurance will have a negative impact on total welfare if the agro-ecosystem is characterized by a high natural insurance function, the private insurance value of agro-biodiversity is low, and its public insurance value is high.

These results are highly relevant for agricultural, environmental and development policy. In so far as it is one aim of development policy to introduce, and improve access to, financial and insurance markets, our analysis shows that such a policy has unambiguously negative implications for agro-biodiversity. Furthermore, our results highlight that properties of agro-ecosystems determine whether welfare increases or decreases under such a policy. Unless a sound agro-biodiversity policy is in place, which should internalize the public benefits of agro-biodiversity for private farmers, improving farmers’ access to financial and insurance markets regardless of agro-ecosystem properties may have adverse welfare effects.

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References


A Appendix

A.1 Proof of Proposition 2

Written down explicitly, the first order conditions (22) and (23) for the interior solution of problem (21), which are obtained as \( \partial U/\partial v = 0 \) and \( \partial U/\partial a = 0 \), are

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \sigma''(v^*) \frac{dv^*}{d\rho} = \frac{1}{2} (1 - a^*)^2 \sigma''(v^*)
\]

\[
\rho (1 - a^*) \sigma'(v^*) = \delta \sigma'(v^*)
\]

(A.1)

(A.2)

Condition (A.2) can be solved to

\[
a^* = 1 - \frac{\delta}{\rho}
\]

(A.3)

Differentiating (A.1) with respect to \( \rho \) and using (A.3) yields

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \sigma''(v^*) \frac{dv^*}{d\rho} = \frac{1}{2} (1 - a^*)^2 \sigma''(v^*)
\]

\[
\frac{dv^*}{d\rho} = - \frac{\delta}{\rho} \frac{1}{2 \rho - \delta} \frac{\sigma''(v^*)}{\sigma''(v^*)} > 0
\]

(A.4)

(A.5)

Differentiating (A.1) with respect to \( \delta \) and using (A.3) yields

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right] \sigma''(v^*) \frac{dv^*}{d\delta} = a^* \sigma''(v^*)
\]

\[
\frac{dv^*}{d\delta} = - \frac{a^*}{2 \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \sigma''(v^*)}
\]

\[
\frac{dv^*}{d\delta} = - \frac{1}{\delta} \frac{\rho - \delta}{\frac{\rho}{2} \sigma''(v^*)} > 0
\]

(A.6)

(A.7)

(A.8)

Differentiating (A.3) with respect to \( \rho \) and \( \delta \) is straightforward and yields expressions for \( da^*/d\rho \) and \( da^*/d\delta \).
A.2 Proof of Proposition 3

Written down explicitly, the first order conditions (28) and (29) for the interior solution of problem (27), which are obtained as \( \partial W/\partial v = 0 \) and \( \partial W/\partial a = 0 \), are

\[-\left[ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right] \sigma^{2''}(\hat{v}) - \frac{\Omega}{2} \Sigma^{2''}(\hat{v}) = c \]  
(A.9)

\[ \rho (1 - \hat{a}) \sigma^2(\hat{v}) = \delta \sigma^2(\hat{v}) \]  
(A.10)

Condition (A.10) can be solved to

\[ \hat{a} = 1 - \frac{\delta}{\rho} \]  
(A.11)

Differentiating (A.9) with respect to \( \rho \) and using (A.11) yields

\[-\left\{ \left[ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right] \sigma^{2''}(\hat{v}) + \frac{\Omega}{2} \Sigma^{2''}(\hat{v}) \right\} \frac{d\hat{v}}{d\rho} = \frac{1}{2} (1 - \hat{a})^2 \sigma^{2'}(\hat{v}) \]  
(A.12)

\[ \frac{d\hat{v}}{d\rho} = \frac{-\frac{1}{2} \delta^2 \sigma^{2'}(\hat{v})}{\delta \left( 1 - \frac{\delta}{2\rho} \right) \sigma^{2''}(\hat{v}) + \frac{\Omega}{2} \Sigma^{2''}(\hat{v})} > 0 \]  
(A.13)

Differentiating (A.9) with respect to \( \Omega \) and using (A.11) yields

\[-\left\{ \left[ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right] \sigma^{2''}(\hat{v}) + \frac{\Omega}{2} \Sigma^{2''}(\hat{v}) \right\} \frac{d\hat{v}}{d\Omega} = \frac{\Omega}{2} \Sigma^{2'}(\hat{v}) \]  
(A.14)

\[ \frac{d\hat{v}}{d\Omega} = \frac{-\frac{1}{2} \Sigma^{2'}(\hat{v})}{\delta \left( 1 - \frac{\delta}{2\rho} \right) \sigma^{2''}(\hat{v}) + \frac{\Omega}{2} \Sigma^{2''}(\hat{v})} > 0 \]  
(A.15)

Differentiating (A.9) with respect to \( \delta \) and using (A.11) yields

\[-\left\{ \left[ \frac{\rho}{2} (1 - \hat{a})^2 + \delta \hat{a} \right] \sigma^{2''}(\hat{v}) + \frac{\Omega}{2} \Sigma^{2''}(\hat{v}) \right\} \frac{d\hat{v}}{d\delta} = \hat{a} \sigma^{2'}(\hat{v}) \]  
(A.16)

\[ \frac{d\hat{v}}{d\delta} = \frac{-\left( 1 - \frac{\delta}{\rho} \right) \sigma^{2'}(\hat{v})}{\delta \left( 1 - \frac{\delta}{2\rho} \right) \sigma^{2''}(\hat{v}) + \frac{\Omega}{2} \Sigma^{2''}(\hat{v})} > 0 \]  
(A.17)

Differentiating (A.10) with respect to \( \rho \) and \( \delta \) is straight forward and yields expressions for \( d\hat{a}/d\rho \), \( d\hat{a}/d\Omega \) and \( d\hat{a}/d\delta \).

A.3 Proof of Proposition 4

(i) From Conditions (A.3) and (A.11) it is apparent that \( a^* = \hat{a} \).

(ii) As \( a^* = \hat{a} \), Conditions (22) and (28) can be interpreted as equations of functions of the single variable \( v \) that determine the levels of \( v^* \) and \( \hat{v} \), respectively. Both conditions have as their right-hand side the constant \( c \), and as their left-hand
side a strictly decreasing function of \(v\), so that \(v^*\) and \(\hat{v}\) are uniquely determined. As the term \(V_{\text{pub}}(v) = -\frac{\Omega}{2} \Sigma^2(v)\) is strictly positive for all \(v\), the left-hand side of Condition (28) is strictly greater than the left-hand side of Condition (22) for all \(v\). As a result the value of \(v\) that equates the left-hand side with the right-hand side is strictly greater for Condition (28) than for Condition (22), i.e. \(\hat{v} > v^*\).

(iii) \(\hat{W} \geq W^*\) by definition of the efficient allocation as the allocation that maximizes \(W\). Strict inequality follows from strict concavity of \(W\) in \(\hat{v}\) and \(\hat{v} > v^*\).

### A.4 Proof of Proposition 5

The first order conditions for the interior solution of Problem (36), which are obtained as \(\partial U/\partial v = 0\) and \(\partial U/\partial a = 0\), are

\[
- \left[ \frac{\rho}{2} (1 - a^*)^2 + \delta \sigma^2(v^*) \right] + \tau = c \\
a^* = 1 - \frac{\delta}{\rho}
\]

(A.18)  
(A.19)

Comparison of Condition (A.18) with Condition (A.9) reveals that

\[v^* = \hat{v} \quad \text{for} \quad \tau = \hat{\tau} = -\frac{\Omega}{2} \Sigma^2(\hat{v})\]  
(A.20)

Employing results (A.13), (A.15) and (A.17), the comparative statics of \(\hat{\tau}\) are

\[
\frac{d\hat{\tau}}{d\Omega} = -\frac{1}{2} \Sigma^2(\hat{v}) \frac{d\hat{v}}{d\Omega} < 0 \\
\frac{d\hat{\tau}}{d\rho} = \frac{\Omega}{2} \Sigma^2(\hat{v}) \frac{d\hat{v}}{d\rho} < 0 \\
\frac{d\hat{\tau}}{d\delta} = -\frac{\Omega}{2} \Sigma^2(\hat{v}) \frac{d\hat{v}}{d\delta} < 0
\]

(A.21)  
(A.22)  
(A.23)

### A.5 Proof of Proposition 6

Differentiating \(W^*\) (Equation 39) with respect to \(\delta\) yields

\[
\frac{dW^*}{d\delta} = -a^* \sigma^2(v^*) - \frac{\Omega}{2} \Sigma^2(v^*) \frac{dv^*}{d\delta}.
\]

(A.24)

So,

\[
\frac{dW^*}{d\delta} \lesssim 0 \quad \Leftrightarrow \quad -\frac{\Omega}{2} \Sigma^2(v^*) \frac{dv^*}{d\delta} \lesssim a^* \sigma^2(v^*)
\]

(A.25)
Employing (A.7), (16) and (20), this condition can be expressed explicitly as

\[-\frac{\Omega^2}{2} \Sigma^{2'}(v^*) \frac{dv^*}{d\delta} \leq a^* \sigma^2(v^*) \]  \hfill (A.26)

\[-\frac{\Omega^2}{2} \Sigma^{2'}(v^*) \leq - \left( \frac{\rho}{2} (1 - a^*)^2 + \delta a^* \right) \frac{\sigma^2(v^*) \sigma^{2''}(v^*)}{\sigma^2(v^*)} \]  \hfill (A.27)

\[V^{pub}(v^*) \leq \left( V^v(v^*, a^*) - \delta a^* \sigma^{2'}(v^*) \right) \frac{\sigma^2(v^*) \sigma^{2''}(v^*)}{[\sigma^2(v^*)]^2} \]  \hfill (A.28)

A.6 Proof of Proposition 7

Differentiating the welfare loss (Equation 43) and using \(a^* = \hat{a}^*\) (Proposition 4) yields

\[\frac{d}{d\delta} \left( \hat{W} - W^* \right) = a^* \left[ \sigma^2(v^*) - \sigma^2(\hat{v}) \right] + \frac{\Omega^2}{2} \Sigma^{2'}(v^*) \frac{dv^*}{d\delta} \]  \hfill (A.29)

Employing (A.7), (16) and (20), one thus has

\[\frac{d}{d\delta} \left( \hat{W} - W^* \right) \approx 0 \quad \Leftrightarrow \quad \frac{d}{d\delta} \left( \hat{W} - W^* \right) \leq a^* \left[ \sigma^2(v^*) - \sigma^2(\hat{v}) \right] \]  \hfill (A.30)

\[-\frac{\Omega^2}{2} \Sigma^{2'}(v^*) \frac{dv^*}{d\delta} \approx \frac{\Omega^2}{2} \Sigma^{2'}(v^*) \frac{dv^*}{d\delta} \leq a^* \left[ \sigma^2(v^*) - \sigma^2(\hat{v}) \right] \]  \hfill (A.31)

\[V^{pub}(v^*) \approx \left( V^v(v^*, a^*) - \delta a^* \sigma^{2'}(v^*) \right) \frac{\sigma^2(v^*) \sigma^{2''}(v^*)}{[\sigma^2(v^*)]^2} \]  \hfill (A.32)

\[V^{pub}(v^*) \approx \left( V^v(v^*, a^*) - \delta a^* \sigma^{2'}(v^*) \right) \left( 1 - \frac{\sigma^2(\hat{v})}{\sigma^2(v^*)} \right) \frac{\sigma^2(v^*) \sigma^{2''}(v^*)}{[\sigma^2(v^*)]^2} \]  \hfill (A.33)
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