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by

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# Irreversibility, ignorance, and the intergenerational equity-efficiency trade-off

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**Abstract:** We demonstrate the existence of an intergenerational equity-efficiency trade-off in policy-making that aims at Pareto-efficiency across generations and sustainability, i.e. non-decreasing utility over time. Our model includes two salient characteristics of sustainability problems and policy: (i) *temporal irreversibility*, i.e. the inability to revise one's past actions; (ii) *closed ignorance*, i.e. future consequences of present actions in human-environment systems may be “unknown unknowns”. If initially unforeseen sustainability problems become apparent and policy is enacted after irreversible actions were taken, policy-making faces a fundamental trade-off between intergenerational Pareto-efficiency and sustainability.

**JEL-Classification:** D3, H23, Q01, Q38, Q56

**Keywords:** climate change, closed ignorance, intergenerational equity-efficiency trade-off, irreversibility, Pareto-efficiency, sustainability

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# 1 Introduction

There is an increasingly alarming development in fundamental environmental indicators such as biodiversity, climate change and non-renewable resource scarcity (e.g. Millenium Ecosystem Assessment 2005, IPCC 2007, UNEP 2007). The prospect that many of the damages inflicted on the environment by today's economic actions will cause problems in the future has intensified the discussion on intergenerational justice, and has led to invoking sustainability as a societal goal in the design of policy. Using the words of the Brundtland Commission's definition of sustainable development (WCED 1987), many fear that the current economic development reduces the ability of future generations to meet their needs. Against this background of an apparently unsustainable situation, sustainability policy has to achieve an efficient and intergenerationally just allocation of resources and their benefits.

A different, but similar, attempt to achieve justice *within* a generation – *intragenerational equity* – in social policy is known to be subject to the equity-efficiency trade-off. This means that the quest for equal utility levels (equity) incurs a Pareto-inefficient allocation (Putterman et al. 1998). With this trade-off in mind, the welfare state was put under close scrutiny by economists who have studied the existence and different mechanisms of this trade-off. Examples include the discussion on the optimal income tax (Mirrlees 1971) or the effects of prolonged unemployment benefits on the duration of unemployment (Blank 2002, Katz and Meyer 1990). The philosophical underpinnings of the trade-off as well as the different conceptions of equity and efficiency used in the literature are discussed in Le Grand (1990).

With this *intragenerational* trade-off in mind, the question emerges whether there also exists an *intergenerational* equity-efficiency trade-off in the attempt to achieve sustainability. In other words: What is the effect of policies aiming at sustainability on Pareto efficiency? Following the intuition from the Second Welfare Theorem, Howarth and Norgaard (1990, 1992) show that in an overlapping-generations-model both equity and efficiency can be achieved intergenerationally, given a set of public policies such as Pigouvian taxes, intergenerational transfer payments and the assignment of resource rights between generations. In a dynamic setting, Solow (1974) finds, in a growth model with exhaustible resources, that applying the Rawlsian maximin principle (equity) favors constant consumption paths over increasing ones. Yet, in his model this is not necessarily intergenerationally inefficient. Krautkraemer and Batina (1999) find that a non-decreasing-utility constraint in a model with a renewable resource can lead to Pareto-inefficient overaccumulation of the resource. In that case, all generations could be made better off by allowing decreasing utility over time. Gerlagh and Keyzer (2001) compare different policy instruments for the sustainable intergenerational distribution of resources and compare them in terms of Pareto-efficiency. They find that a trust fund, in which all natural resources and ecosystem services are administered that can be produced sustainably, leads to a Pareto improvement compared to a zero-extraction policy. Krysiak (2009) studies the trade-off between sustainability and Pareto-efficiency with respect to uncertain future outcomes and preferences. He finds a trade-off between protecting future individuals from potential harm (sustainability) and thereby abstaining from actions that would have made everyone better-off (efficiency).

In this paper, we investigate how an intergenerational equity-efficiency trade-off in sustainability policy emerges from the genuine character and mechanisms of intergenerational policy-making. Compared to *intragenerational* policy-making, there are two salient characteristics of sustainability problems and policy: (i) temporal irreversibility (Baumgärtner 2005), i.e. the inability to revise one’s past actions; (ii) “closed ignorance” (Faber et al. 1992), i.e. future consequences of present actions may be “unforeseen contingencies” (Dekel et al. 1998), also known as “unknown unknowns” (Rumsfeld 2002).

Our two-non-overlapping-generations model combines the canonical intragenerational leisure-consumption choice from labor economics with a non-renewable natural resource, an intergenerational negative externality from production, and investment into technical progress. Both generations use the stock of the non-renewable resource for production, with the second generation being more productive due to technical progress. The sustainability problem arises from a damage to the second generation from the first generation’s use of the non-renewable resource. Initially, there is closed ignorance about this damage, i.e. it is an “unknown unknown” in the sense that it is unforeseen and the first generation is unaware of its ignorance. The damage only becomes apparent after production by the first generation has irreversibly taken place.

Within this framework we study policy-making by a social planner who pursues the two goals of (1) Pareto-efficiency accross generations and (2) sustainability as non-decreasing utility over time (Howarth 1995). To achieve this, she has two policy instruments at hand: (1) she can assign resource-use rights between generations, and (2) she can oblige the first generation to invest into technical progress, thus enhancing the

productivity of the second generation. Initially, the social planner acts under closed ignorance about the damage, but she can adjust her policy mix after her ignorance has been resolved and the extent of damage has become apparent.

We demonstrate that the social planner faces a fundamental trade-off between intergenerational Pareto-efficiency and sustainability: she can achieve either one of these two goals, but not both, if policy-making is done initially under closed ignorance, and can be adjusted only after irreversible actions were made.

An important case in point is the current discussion in the political realm of “climate justice”, which is concerned with the equitable distribution of the benefits and damages from CO<sub>2</sub>-emissions between developing and industrialized countries as well as between past and future emitters (Neumayer 2000). This concern can be analyzed in our model as an issue of intergenerational distributional equity (sustainability). The first generation in the model corresponds to societies in countries (e.g. in Europe and North America, “historic emitters” for short) that were the main emitter of CO<sub>2</sub> in the past, and, in fact, have emitted more than the atmosphere could absorb, thereby causing climate change. However, the actions of the historic emitters were made without the knowledge of the effects of CO<sub>2</sub> on climate change, i.e. under “closed ignorance”. The second generation in the model corresponds to societies in countries (e.g. China and India, “future emitters” for short) that are industrializing quickly, want to emit more CO<sub>2</sub> over their future development, and now suffer the damages from climate change.<sup>1</sup>

The past CO<sub>2</sub>-emissions of the historic emitters result in climate change that (with-

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<sup>1</sup>The absorption of CO<sub>2</sub> in the atmosphere is quite slow on the relevant time scale, so one can conceptualize the limited capacity of the atmosphere to safely absorb CO<sub>2</sub> as a non-renewable resource.

out compensation) reduces the welfare of future emitters below the level that the historic emitters enjoyed. This is seen by many as an intergenerational distributional inequity that should be corrected by a compensation of the future emitters by the historic emitters. The future emitters are more productive than the historic emitters were at the time of their emission which reduces the necessary level of compensation.

The crucial challenge here is that sustainability policy is being shaped and implemented after historic emissions have already irreversibly taken place as a result of the historic emitter's technology and consumption decisions in the past, e.g. to produce energy mainly from fossil fuels. This means that the historic emitters are now asked to provide a compensation after they have already irreversibly realized their production and emissions, so that they cannot react to current sustainability policy by adjusting their (past) production and emissions. This corresponds to the sustainability policy studied in our model, where the first generation has taken irreversible production decisions under closed ignorance, and is put under an obligation by sustainability policy after that.

For the case of climate justice, our result means that climate policy in the current situation faces a trade-off between intergenerational equity and Pareto-efficiency. In other words, any attempt to achieve climate justice between historic and future emitters necessarily leads to a Pareto-inefficient allocation – and all Pareto-efficient policies will not be equitable.

The paper is organized as follows. Section 2 introduces the model. In Section 3, the normative criteria of sustainability and efficiency are defined within the framework of



the model. In Section 4, the effects of temporal irreversibility and closed ignorance on policy-making are examined. Finally, Section 5 concludes.

## 2 Model

There are two successive, i.e. non-overlapping, generations ( $t=1,2$ ). Both generations have identical preferences over leisure  $L_t$  and consumption  $C_t$  that are represented by a representative individual's continuous, monotonic and quasi-concave utility function

$$U_t = U(C_t, L_t) , \quad (1)$$

which fulfils the Inada conditions  $\lim_{C \rightarrow 0} \partial U / \partial C = \lim_{L \rightarrow 0} \partial U / \partial L = \infty$  and  $U(0, L_t) = 0$ .

Both generations are endowed with the same time budget  $Z$  which can either be enjoyed as leisure  $L_t$  with  $0 \leq L_t \leq Z$  or be used as labor time  $Z - L_t$  in the production of some all-purpose intermediate good.

Total endowment with the natural non-renewable resource is  $\bar{R}$ , with  $R_t$  denoting the amount used by generation  $t$  for production. Production of the intermediate good  $Y_t$  in each period depends on the inputs of labor  $Z - L_t$  and the natural resource  $R_t$ , as well as on the productivity  $w_t$  in that period,

$$Y_t = w_t F(Z - L_t, R_t) , \quad (2)$$

where the production function  $F$  exhibits positive and decreasing marginal products of both labor and resource input, and both labor and the resource are essential for production,  $F(0, R_t) = F(Z - L_t, 0) = 0$ .

The intermediate good thus produced in  $t = 1$  can either be directly consumed, or it may be invested into technical progress:

$$Y_1 = C_1 - T , \quad (3)$$

where  $T$  increases the rate of factor-neutral technical progress:

$$\begin{aligned} \frac{w_2}{w_1} &= \gamma(T) \quad \text{with} \quad 1 \leq \gamma(0) < \gamma^s , \\ \gamma'(T) &> 0 \quad \text{and} \quad \gamma''(T) \leq 0 \quad \text{for all } T \geq 0 . \end{aligned} \quad (4)$$

The rate of autonomous technical progress,  $\gamma(0)$ , is bounded by some maximal value  $\gamma^s$  so as to prevent a trivial automatic solution to the sustainability problem discussed here. Generation 2, being the last generation, has no option to invest but will devote the entire amount of the intermediate good produced in  $t = 2$  to consumption.

The first generation's use of the resource in production causes damages  $D(R_1)$  to the second generation, i.e. it diminishes the availability of their social product for consumption,

$$C_2 = Y_2 - D(R_1) , \quad (5)$$

with marginal damages being positive and increasing,  $D'(R_1) > 0$  and  $D''(R_1) \geq 0$  for all  $R_1 > 0$ , and total damages  $D(R_1)$  being not too large to prevent a solution to the sustainability problem through investment. Initially, i.e. before any production, uncertainty prevails about these damages as so-called “closed ignorance” (Faber et al. 1992). That is, generation 1 is initially not aware of any potential future damages, and is not even aware of its ignorance, but firmly believes that resource use does not entail any future damages. Thus, future damages are “unforeseen contingencies” (Dekel et al.

1998), or “unknown unknowns” (Rumsfeld 2002). This ignorance is resolved, and the full extent of damages becomes apparent, after production by generation 1 has taken place.

Sustainability comes into play through a social planner who aims at (1) Pareto-efficiency across generations and (2) sustainability in the sense of non-decreasing utility over time (Howarth 1995). She acts during the first generation’s lifetime and shares the same information as the first generation. In order to achieve her two goals, the social planner has two policy instruments at hand: (1) she can restrict resource use of generation  $t$  by an upper limit  $\bar{R}_t$ ; (2) she can oblige generation 1 to invest at least an amount  $\underline{T}$  out of its intermediate product  $Y_1$  into technical progress and, thus, enhance the productivity of generation 2.

The exact time structure is as follows. There are three time stages:  $t = 1a$ ,  $t = 1b$  and  $t = 2$ . Generation 1 lives in the first two of these, while generation 2 lives in the last one. In the first stage  $t = 1a$ , generation 1 chooses its production level of the intermediate good  $Y_1$ , its leisure level  $L_1$ , and the share  $R_1$  of the total resource endowment  $\bar{R}$ , so as to maximize its own expected utility  $U_1$  subject to restrictions imposed by technology and policy. At this stage, the social planner may restrict the resource use  $R_1$  and make generation 1 plan with a minimal investment of  $\underline{T}$ . Production takes place in this stage, so that the inputs are irreversibly sunk, but as production takes time, the output is not turned out before the next stage. In the second stage  $t = 1b$ , output  $Y_1$  of the intermediate good becomes available for use. At the same time, uncertainty is resolved and the future damages  $D(R_1)$  from using the resource in

production become fully apparent. In reaction to this information, the social planner may want to adjust her policy at this stage. As production by generation 1 has already taken place and resources  $R_1$  and labor time  $Z - L_1$  are irreversibly sunk, she cannot revise the restriction on resource use any more, but she can now adjust only her second policy instrument and enforce generation 1 to invest a higher amount  $\underline{T}$  out of its intermediate good into technical progress, thereby reducing generation 1's consumption  $C_1$  and increasing generation 2's consumption  $C_2$ . Generation 1 cannot revise its original production decision anymore at this stage, as the inputs are already irreversibly sunk. In the third stage  $t = 2$ , generation 2 chooses its leisure level  $L_2$  and its production of intermediate goods  $Y_2$ , which are entirely consumed in this same stage, with a reduction due to the damages caused by generation 1's resource use.

### 3 Sustainability and efficiency

Throughout this analysis we understand the terms “sustainability” and “efficiency” as follows. Sustainability is defined as non-decreasing utility over time. This criterion fits the context of the model as both generations have identical preferences, which allows intergenerational comparability.

**Definition 1** (Sustainability)

An intergenerational allocation  $(C_1, L_1, R_1, T, C_2, L_2, R_2)$  is called *sustainable* if and only if it is feasible and

$$U(C_2, L_2) \geq U(C_1, L_1) , \tag{6}$$

where the minimum requirement is that  $U(C_2, L_2) = U(C_1, L_1)$ .

Efficiency is defined as ex-ante Pareto-efficiency, which means that one cannot make a generation better-off without making the other worse-off before uncertainty is resolved. As always under uncertainty, it may be that an allocation which is considered to be efficient ex ante, i.e. while uncertainty still prevails, turns out to be actually inefficient ex post, i.e. after uncertainty has been resolved. Therefore, the guiding principle for policy-making at a point in time when uncertainty still prevails should be ex-ante efficiency.

**Definition 2** (Pareto-efficiency)

A feasible intergenerational allocation  $(C_1, L_1, R_1, T, C_2, L_2, R_2)$  is called *Pareto-efficient* if and only if there exists no other feasible intergenerational allocation  $(C'_1, L'_1, R'_1, T', C'_2, L'_2, R'_1)$  for which  $U(C'_t, L'_t) \geq U(C_t, L_t)$  for  $t = 1, 2$  and  $U(C'_t, L'_t) > U(C_t, L_t)$  for at least one  $t$ .

With this definition, Pareto-efficient allocations are characterized as follows.

**Lemma 1**

A feasible intergenerational allocation  $(C_1, L_1, R_1, T, C_2, L_2, R_2)$  is Pareto-efficient if and only if it meets the following conditions:

$$\frac{\partial U(C_1 - T, L_1) / \partial L}{\partial U(C_1 - T, L_1) / \partial C} = -w_1 \frac{\partial F(Z - L_1, R_1)}{\partial L}, \quad (7)$$

$$\frac{\partial U(C_2, L_2) / \partial L}{\partial U(C_2, L_2) / \partial C} = -w_1 \gamma(T) \frac{\partial F(Z - L_2, R_2)}{\partial L}, \quad (8)$$

$$w_1 \gamma'(T) F(Z - L_2, R_2) \frac{\partial F(Z - L_1, R_1)}{\partial R} = \gamma(T) \frac{\partial F(Z - L_2, R_2)}{\partial R}, \quad (9)$$

$$U(w_1 \gamma(T) F(Z - L_2, R_2), L_2) = \bar{U}, \quad (10)$$

$$\bar{R} - R_1 - R_2 = 0, \quad (11)$$

where  $\bar{U} \geq 0$  is an intergenerational distribution parameter which can take on infinitely many values.

*Proof.* See Appendix A.1. □

The intergenerational distribution parameter  $\bar{U}$  can attain any value between 0 (all potential utility in this system is with generation 1, none with generation 2) and  $+\infty$  (all potential utility in this system is with generation 2, none with generation 1), so that there exist infinitely many Pareto-efficient allocations satisfying this characterization. Condition (7) states that generation 1's marginal rate of substitution between leisure and consumption must equal this generation's marginal productivity of another hour of work (intragenerational efficiency in  $t = 1$ ). Condition (8) states the same for generation 2 (intragenerational efficiency in  $t = 2$ ). Condition (9) states that the marginal gain in production for generation 2 from either of the following two alternative uses of the resource should be equal: (LHS) giving one additional marginal unit of the resource to generation 1 as input into production, and then investing the entire additional amount of the intermediate good thus produced into technical progress; (RHS) giving one additional marginal unit of the resource directly to generation 2 as input into their production (intergenerational efficiency). Conditions (10) and (11) state that the constraints for generation 2's utility level exceeding  $\bar{U}$  and total resource use not exceeding  $\bar{R}$  must hold with equality. As there is closed ignorance about the damage to generation 2 from generation 1's resource use is completely unknown ex ante, this damage does not show up in Conditions 7–11 which characterize ex-ante efficiency.

One can illustrate efficient allocations through a Pareto-frontier  $U_2(U_1)$  in utility space (cf. Figure 1). Some general statements about its shape can be made within the model. The maximal achievable utility level for generation 1 is achieved by not investing

anything in technical progress  $\underline{T} = 0$  which results in  $U_1^{max}$  and  $U_2^{min} = 0$  due to the essentiality of the resource in production and the essentiality of consumption for utility. The reverse is the policy that maximizes the utility level of the second generation  $U_2^{max}$ . For this an efficient amount of the resource is to be allocated to the second generation and all of the first generation's production has to be invested in technical progress which leads to  $U_1^{min} = 0$ . Hence, as  $U_2(U_1^{min}) = U_2^{max}$  and  $U_2(U_1^{max}) = U_2^{min}$  and if  $U_t^{min} = 0$  the Pareto-frontier starts from the axes. Furthermore, the Pareto-frontier is negatively inclined as reducing resource use of generation 1 decreases  $U_1$  and increases  $U_2$  and reducing intermediate production of generation 1 through investment decreases  $U_1$  and increases  $U_2$ . Therefore, the Pareto-frontier is negatively inclined. In the same way, sustainability can be illustrated by a sustainability threshold as a 45°-line from the axes. This illustration will be used below to analyze the results of the model.

## 4 Results

In this section, the existence of a sustainability-efficiency trade-off is demonstrated which emerges as the combined effect of temporal irreversibility and closed ignorance about future damages. Thereby, we follow the time structure laid out in Section 2. In the first stage  $t = 1a$ , the social planner devises a policy mix of restrictions on resource use and minimal investment into technical progress, that should lead to an intergenerational allocation that is both Pareto-efficient and sustainable. In the second stage  $t = 1b$ , the damages from resource use  $D(R_1)$  become apparent, and for the revision of the policy mix a trade-off between efficiency and sustainability emerges: there exists an

adjustment of the policy mix that ensures sustainability but not efficiency, and there exists an adjustment of the policy mix that ensures efficiency but not sustainability, but there no longer exists a policy mix that ensures both efficiency and sustainability.

In the first stage  $t = 1a$ , the social planner devises a sustainable and efficient policy. She uses the two instruments of restriction of resource use and investment in technical progress to ensure the attainment of the two goals of efficiency and sustainability.

**Lemma 2** (Sustainable and efficient policy)

At time  $t = 1a$ , there exists a unique policy mix  $(\bar{R}_1^a, \bar{R}_2^a, \underline{T}^a)$  that leads to an allocation  $(\tilde{C}_1, \tilde{L}_1, \tilde{R}_1, \tilde{T}, \tilde{C}_2, \tilde{L}_2, \tilde{R}_2)$  which is sustainable and Pareto-efficient. This allocation is characterized by the following necessary and sufficient conditions:

$$\frac{\partial U(\tilde{C}_1 - \underline{T}^a, \tilde{L}_1)/\partial L}{\partial U(\tilde{C}_1 - \underline{T}^a, \tilde{L}_1)/\partial C} = -w_1 \frac{\partial F(Z - \tilde{L}_1, \bar{R}_1^a)}{\partial L}, \quad (12)$$

$$\frac{\partial U(\tilde{C}_2, \tilde{L}_2)/\partial L}{\partial U(\tilde{C}_2, \tilde{L}_2)/\partial C} = -w_1 \gamma(\underline{T}^a) \frac{\partial F(Z - \tilde{L}_2, \bar{R}_2^a)}{\partial L}, \quad (13)$$

$$w_1 \gamma'(\underline{T}^a) F(Z - \tilde{L}_2, \bar{R}_2^a) \frac{\partial F(Z - \tilde{L}_1, \bar{R}_1^a)}{\partial R} = \gamma(\underline{T}^a) \frac{\partial F(Z - \tilde{L}_2, \bar{R}_2^a)}{\partial R}, \quad (14)$$

$$\bar{R} - \bar{R}_1^a - \bar{R}_2^a = 0, \quad (15)$$

$$U(w_1 F(Z - \tilde{L}_1, \bar{R}_1^a) - \underline{T}^a, \tilde{L}_1) = U(w_1 \gamma(\underline{T}^a) F(Z - \tilde{L}_2, \bar{R} - \bar{R}_1^a), \tilde{L}_2) \quad (16)$$

*Proof.* See Appendix A.2. □

Under this policy the first generation makes its leisure-consumption choice  $\tilde{C}_1, \tilde{L}_1$  as in Equation (12). It also chooses to fully use the resource  $\tilde{R}_1 = \bar{R}_1^a$  as resource use carries no costs and not to provide any transfer above the minimum level  $\tilde{T} = \underline{T}^a$  as it is not altruistic. The social planner assumes that the second generation makes its decision on resource use, leisure and consumption  $\tilde{C}_2, \tilde{L}_2, \tilde{R}_2$  as in Equation (13). With a



combination of the two policy instruments that satisfies Equations (14) and (15) she can achieve an allocation that is both Pareto-efficient and sustainable as in Equations (12) – (15). In climate policy this refers to the policy under which extraction, production and consumption of the historic emitters took place. The use of fossil fuels was thought to be harmless as the effects of CO<sub>2</sub> on the atmosphere were unknown unknowns. Future emitters were thought to be well-off due to the effects of technical progress and the ability to emit some more CO<sub>2</sub>.

However, with the increasing evidence that climate change was existing and of relevant magnitude with the first report of the IPCC (1990) the damages of CO<sub>2</sub> became apparent. Therefore, in the second stage  $t = 1a$  the damages from resource use  $D(R_1)$  become apparent and the trade-off between efficiency and sustainability emerges. It is clear that no adaption of policy to the new findings would result in an unsustainable allocation.

$$U(w_1 F(Z - \tilde{L}_1, \bar{R}_1^a) - \underline{T}^a, \tilde{L}_1) > U(w_1 \gamma(\underline{T}^a) F(Z - \tilde{L}_2, \bar{R}_2^a) - D(\bar{R}_1^a), \tilde{L}_2) \quad (17)$$

The social planner must therefore adapt its policy to these new findings if she wants to be sustainable. However, the initial production decision of the first generation  $\tilde{Y}_1, \tilde{L}_1, \bar{R}_1^a$  has been made in Equation (12) and inputs are irreversibly sunk. She still could restrict the second generation's resource use  $\bar{R}_2$  even further, but this would make the attainment of sustainability even more difficult. The only viable option to achieve sustainability is therefore to increase the minimum investment in technical progress  $\underline{T}^b > \underline{T}^a$ .

**Proposition 1** (Sustainability-efficiency trade-off)

There exists a trade-off between sustainability and efficiency for policy-making at time

$t = 1b$ . At this stage,

- (i) there exists no feasible policy mix  $(\bar{R}_1^b, \bar{R}_2^b, \underline{T}^b)$  that yields an allocation that is both Pareto-efficient and sustainable, but
- (ii) there exists a unique level of minimum investment  $\underline{T}^{bs}$  that yields an allocation  $(\hat{C}_1, \hat{L}_1, \hat{R}_1, \hat{T}, \hat{C}_2, \hat{L}_2, \hat{R}_2)$  that is sustainable but not Pareto-efficient, and
- (iii) there exists another unique level of minimum investment  $\underline{T}^{be} = \underline{T}^a$  that yields an allocation  $(C_1^*, L_1^*, R_1^*, T^*, C_2^*, L_2^*, R_2^*)$  that is Pareto-efficient but not sustainable.

*Proof.* See Appendix A.3. □

The intuition behind this result is as follows. Any investment  $\underline{T}^b \neq \underline{T}^a$  does not meet Condition (12) for efficiency as it has to change an irreversible decision in retrospect. This corresponds to the current situation in climate policy where damages from CO<sub>2</sub> emissions were unknown initially. Here, historic emitters have to provide compensation after they have made their production decision and cannot change their leisure or emission levels in retrospect.

By investing more in technical progress  $\underline{T}^{bs}$  at time  $t = 1b$  the social planner can achieve a sustainable allocation as in Proposition 1 (ii). The first generation's production decision from Equation (12) is irreversible  $\hat{Y}_1 = \tilde{Y}_1, \hat{L}_1 = \tilde{L}_1, \hat{R}_1 = \bar{R}_1^a$ , but more of the intermediate product  $\hat{Y}_1$  can be used for investment. Under this policy the second generation will again use its resource endowment completely  $\hat{R}_2 = \bar{R}_2^b$  and make its leisure-consumption decision  $\hat{C}_2, \hat{L}_2$  with respect to the higher investment  $\underline{T}^{bs}$ . With

this higher investment sustainability is achieved despite the damages:

$$U(w_1 F(Z - \hat{L}_1, \bar{R}_1^{bs}) - \underline{T}^{bs}, \hat{L}_1) = U(w_1 \gamma(\underline{T}^{bs}) F(Z - \hat{L}_2, \bar{R} - \bar{R}_1^{bs}) - D(\bar{R}_1^{bs}), \hat{L}_2) \quad (18)$$

As the investment has to be increased in retrospect  $\underline{T}^{bs} > \underline{T}^a$  Condition (12) is violated.

In climate policy this corresponds to an investment in technical progress which ensures the future emitters the same welfare as historic emitters enjoyed. This investment would exceed the one originally assumed necessary for sustainability.

As noted in Proposition 1 (iii) a Pareto-efficient and unsustainable allocation  $(C_1^*, L_1^*, R_1^*, T^*, C_2^*, L_2^*, R_2^*)$  can be achieved by not investing more in technical progress  $\underline{T}^{bw} = \underline{T}^a$ . In climate policy this corresponds to a situation where no additional investment in technical progress is made to benefit future emitters.

Despite the described trade-off in policy-making, there exists, in principle, a feasible allocation that is both Pareto-efficient and sustainable.

**Proposition 2** (Bliss)

Among all feasible Pareto-efficient allocations there uniquely exists one that is sustainable.

*Proof.* See Appendix A.4. □

This result shows that the trade-off at  $t = 1b$  between equity and efficiency (Proposition 1) is due to the characteristics of temporal irreversibility and closed ignorance.

The relationship of the two policies to sustainability and efficiency is illustrated in Figure 1. Here the efficient policy allocation lies on the Pareto frontier which shows all Pareto-efficient allocations under consideration of the damages  $D(R_1)$ , but below the

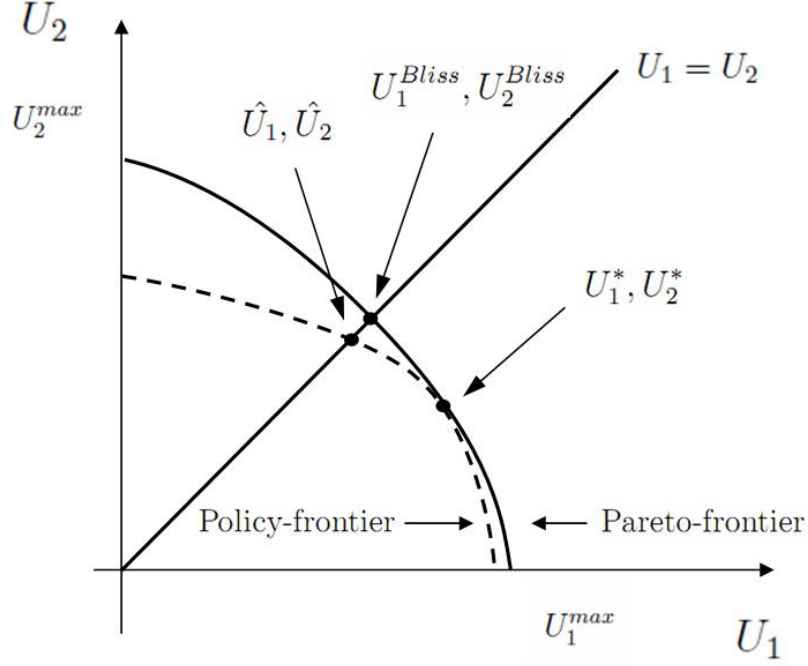


Figure 1: Sustainable distributions (solid line), Pareto-frontier (solid curve) and policy frontier (dashed curve) in utility space, with Pareto-efficient policy (\*) and sustainable policy (^). No feasible policy can achieve the feasible allocation that is both Pareto-efficient and sustainable (“Bliss”).

sustainability threshold that indicates  $U_1 = U_2$ . The sustainability policy allocation is below the Pareto frontier, but lies on the sustainability threshold. At the intersection of these two lies the Pareto-efficient and sustainable allocation  $U_1^{Bliss}, U_2^{Bliss}$ . The trade-off between sustainability and efficiency consists here in the impossibility to reach both goals at the same time, i.e. one must choose between the Pareto-efficient allocation at  $U_1^*, U_2^*$  and the sustainable allocation at  $\hat{U}_1, \hat{U}_2$ . The combination of these goals that the social planner can achieve at  $t = 1b$  lie on the dotted policy frontier.

## 5 Conclusion

We have studied the question of whether there exists a mechanism genuine to intergenerational policy-making that causes an intergenerational equity-efficiency trade-off. We found that sustainability policy that acts under a combination of temporal irreversibility and closed ignorance faces such a trade-off between Pareto-efficiency across generations and intergenerational equity.

This result is relevant for current climate policy. Policies that want to achieve sustainability after damages were initially unknown (closed ignorance) must respect that past actions cannot be undone (temporal irreversibility), and that redistribution must therefore face a trade-off between efficiency and sustainability. For the case of climate justice – where climate policy is enacted after the build-up of a carbon-dependant economy – this means that there is a trade-off between equity and Pareto-efficiency among historic and future emitters. Policymakers therefore need to be aware of the fact that pursuing sustainability as the overriding priority sacrifices Pareto-efficiency, and that prudent policymaking requires a prior debate on how to balance these two conflicting goals.

Another conclusion concerns the timing of sustainability policy. As the equitable solution of an already advanced sustainability problem is Pareto-inefficient due to temporal irreversibility, this provides an argument for enacting sustainability policy sooner rather than later. This is in contrast to the intuition from the well-known quasi-option value of waiting under circumstances of uncertainty (Arrow and Fisher 1974, Henry 1974, Hanemann 1989) that has been cited to justify a procrastination of climate policy.

Yet, there is still need to analyze different instruments in sustainability policy with respect to their effect on Pareto-efficiency (as in Gerlagh and Keyzer 2001). Describing and quantifying the trade-offs between sustainability and Pareto-efficiency is necessary for the design of concrete policies. After all, we do not want to pay more for sustainability than necessary.

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## Appendix

### A.1 Proof of Lemma 1

A feasible Pareto-efficient allocation, sensu Definition 2, is the solution to the optimization problem

$$\begin{aligned}
& \max_{C_1, L_1, R_1, T, C_2, L_2, R_2} U(C_1, L_1) \quad \text{s.t.} \quad U(C_2, L_2) \geq \bar{U} , \\
& Y_1 = w_1 F(Z - L_1, R_1), \quad C_1 = Y_1 - T , \\
& Y_2 = w_1 \gamma(T) F(Z - L_2, R_2), \quad C_2 = Y_2 , \\
& \bar{R} \geq R_1 + R_2, \\
& 0 \leq L_1 \leq Z, \quad 0 \leq L_2 \leq Z , \tag{A.19}
\end{aligned}$$

Obviously,  $\bar{R} > R_1 + R_2$  cannot be Pareto-efficient, but the resource must be used completely for Pareto-efficiency,  $\bar{R} = R_1 + R_2$ . As the utility function satisfies an Inada condition for consumption, and labor is essential in the production of consumption, one has  $L_t < Z$  for  $t = 1, 2$ . This leads to the Lagrangian:

$$\begin{aligned}
\mathfrak{L} = & U(w_1 F(Z - L_1, R_1) - T, L_1) + \lambda_1 (U(w_1 \gamma(T) F(Z - L_2, R_2), L_2) - \bar{U}) \\
& + \lambda_2 (\bar{R} - R_1 - R_2) . \tag{A.20}
\end{aligned}$$

Obviously, in the optimal solution the constraint  $U(C_2, L_2) \geq \bar{U}$  must hold with equality. Because the utility function satisfies an Inada condition for  $L$ , the efficient allocation

cannot be  $L_t = 0$  (for  $t = 1, 2$ ). The necessary first order conditions then are:

$$\frac{\partial \mathfrak{L}}{\partial L_1} = \frac{\partial U(C_1 - T, L_1)}{\partial C} w_1 \frac{\partial F(Z - L_1, R_1)}{\partial L} + \frac{\partial U(C_1 - T, L_1)}{\partial L} = 0 , \quad (\text{A.21})$$

$$\frac{\partial \mathfrak{L}}{\partial L_2} = \lambda_1 \left[ \frac{\partial U(C_2, L_2)}{\partial C} w_1 \gamma(T) \frac{\partial F(Z - L_2, R_2)}{\partial L} + \frac{\partial U(C_2, L_2)}{\partial L} \right] = 0 , \quad (\text{A.22})$$

$$\frac{\partial \mathfrak{L}}{\partial R_1} = \frac{\partial U(C_1 - T, L_1)}{\partial C} w_1 \frac{\partial F(Z - L_1, R_1)}{\partial R} - \lambda_2 = 0 , \quad (\text{A.23})$$

$$\frac{\partial \mathfrak{L}}{\partial R_2} = \lambda_1 \frac{\partial U(C_2, L_2)}{\partial C} w_1 \gamma(T) \frac{\partial F(Z - L_2, R_2)}{\partial R} - \lambda_2 = 0 , \quad (\text{A.24})$$

$$\frac{\partial \mathfrak{L}}{\partial T} = -\frac{\partial U(C_1 - T, L_1)}{\partial C} + \lambda_1 \frac{\partial U(C_2, L_2)}{\partial C} w_1 \gamma'(T) F(Z - L_2, R_2) = 0 , \quad (\text{A.25})$$

$$\frac{\partial \mathfrak{L}}{\partial \lambda_1} = U(w_1 \gamma(T) F(Z - L_2, R_2), L_2) - \bar{U} = 0 , \quad (\text{A.26})$$

$$\frac{\partial \mathfrak{L}}{\partial \lambda_2} = \bar{R} - R_1 - R_2 = 0 . \quad (\text{A.27})$$

As  $\lambda_1$  is the Lagrange multiplier for the constraint  $U(C_2, L_2) \geq \bar{U}$ , its optimal value measures by how much the optimal value of the objective function  $U(C_1, L_1)$  increases when the constraint  $\bar{U}$  is relaxed by one marginal unit. In our model, this is strictly positive,  $\lambda_1 > 0$ , so that in Equation (A.22) the term in brackets must vanish. Solving Equations (A.21) and (A.22) for  $(\partial U / \partial L) / (\partial U / \partial C)$ , solving Equations (A.23) and (A.24) for  $\lambda_2$  and eliminating  $\lambda_2$  by equating the two, and eliminating  $\lambda_1$  from the resulting equation with Equation (A.25), one arrives at Equations (7)–(11). With the curvatures assumed for the functions  $U$  and  $F$ , the Lagrangian (A.20) is strictly concave, so that these conditions are also sufficient.

## A.2 Proof of Lemma 2

At time  $t = 1a$ , the social planner chooses a policy  $(\bar{R}_1^a, \bar{R}_1^a, \underline{T}^a)$ . Under this policy the first generation optimizes:

$$\begin{aligned}
\max_{C_1, L_1, R_1, T} U(C_1, L_1) \quad \text{s.t.} \quad & Y_1 = w_1 F(Z - L_1, R_1), \\
& C_1 = Y_1 - T, \\
& \underline{T}^a \leq T \leq Y_1, \\
& \bar{R}_1^a \geq R_1, \\
& 0 \leq L_1 \leq Z
\end{aligned} \tag{A.28}$$

which yields  $(\tilde{C}_1, \tilde{L}_1, \tilde{R}_1, \tilde{T})$  as solution. As the first generation derives no utility from altruism it will not make any investment above the minimum level  $\tilde{T} = \underline{T}^a$ . It will use the resource completely  $\tilde{R}_1 = \bar{R}_1^a$  as there are no opportunity costs of resource use. As  $U$  satisfies an Inada condition for  $C$ , and labor is essential in the production of consumption, one has  $L_1 < Z$ . This leads to the following, simpler optimization problem:

$$\max_{L_1} U(w_1 F(Z - L_1, \bar{R}_1^a) - \underline{T}^a, L_1), \tag{A.29}$$

with the first-order condition:

$$\frac{dU}{dL_1} = \frac{\partial U(\tilde{C}_1 - \underline{T}^a, \tilde{L}_1)}{\partial C} w_1 \frac{\partial F(Z - \tilde{L}_1, \bar{R}_1^a)}{\partial L} + \frac{\partial U(\tilde{C}_1 - \underline{T}^a, \tilde{L}_1)}{\partial L} = 0. \tag{A.30}$$

As the damages are not known initially, the social planner assumes that the second generation maximizes the following:

$$\begin{aligned}
\max_{C_2, L_2, R_2} U(C_2, L_2) \quad \text{s.t.} \quad & Y_2 = w_1 \gamma(\tilde{T}) F(Z - L_2, R_2), \\
& C_2 = Y_2, \\
& \bar{R}_2^a \geq R_2, \\
& 0 \leq L_2 \leq Z,
\end{aligned} \tag{A.31}$$

which yields  $(\tilde{C}_2, \tilde{L}_2, \tilde{R}_2)$  as solution. It will also use the resource completely  $\tilde{R}_2 = \bar{R}_2^a$  as there are no opportunity costs. As  $U$  satisfies an Inada condition for  $C$ , and labor is essential in the production of consumption, one has  $L_2 < Z$ . This leads to the following, simpler optimization problem:

$$\max_{L_2} U(w_1 \gamma(\tilde{T}) F(Z - L_2, \bar{R}_2^a), L_2), \tag{A.32}$$

with the first-order condition:

$$\frac{dU}{dL_2} = \frac{\partial U(\tilde{C}_2, \tilde{L}_2)}{\partial C} w_1 \gamma(\tilde{T}) \frac{\partial F(Z - \tilde{L}_2, \bar{R}_2^a)}{\partial L} + \frac{\partial U(\tilde{C}_2, \tilde{L}_2)}{\partial L} = 0. \tag{A.33}$$

The social planner chooses a policy  $(\bar{R}_1^a, \bar{R}_2^a, \underline{T}^a)$  so that the resulting allocation  $(\tilde{C}_1, \tilde{L}_1, \tilde{R}_1, \tilde{T}, \tilde{C}_2, \tilde{L}_2, \tilde{R}_2)$  is efficient, i.e. according to Lemma 1 it satisfies Conditions (7)–(11), and sustainable, i.e. according to Definition 1 it fulfills Condition (6). For that sake, she takes into account the individually optimizing reactions of both generations to her policy (Conditions A.30 and A.33) which – when the policy mix is appropriate – ensure intragenerational efficiency in both generation’s leisure-consumption allocation (Conditions 7 and 8), she uses a feasible and efficient distribution of the total

resource endowment (Condition 11), she takes into account intergenerational efficiency (Condition 9) and sustainability,  $U(C_2, L_2) = U(C_1, L_1)$  (Definition 1), which selects one particular distribution of utility among generations out of the infinitely many possible ones that are all Pareto-efficient according to Condition (10). All taken together, she arrives at the following conditions, which are equivalent to Conditions (12)–(16):

$$\frac{\partial U(\tilde{C}_1 - \underline{T}^a, \tilde{L}_1)/\partial L}{\partial U(\tilde{C}_1 - \underline{T}^a, \tilde{L}_1)/\partial C} = -w_1 \frac{\partial F(Z - \tilde{L}_1, \bar{R}_1^a)}{\partial L}, \quad (\text{A.34})$$

$$\frac{\partial U(\tilde{C}_2, \tilde{L}_2)/\partial L}{\partial U(\tilde{C}_2, \tilde{L}_2)/\partial C} = -w_1 \gamma(\underline{T}^a) \frac{\partial F(Z - \tilde{L}_2, \bar{R}_2^a)}{\partial L}, \quad (\text{A.35})$$

$$w_1 \gamma'(\underline{T}^a) F(Z - \tilde{L}_2, \bar{R}_2^a) \frac{\partial F(Z - \tilde{L}_1, \bar{R}_1^a)}{\partial R} = \gamma(\underline{T}^a) \frac{\partial F(Z - \tilde{L}_2, \bar{R}_2^a)}{\partial R}, \quad (\text{A.36})$$

$$\bar{R} - \bar{R}_1^a - \bar{R}_2^a = 0, \quad (\text{A.37})$$

$$U(w_1 F(Z - \tilde{L}_1, \bar{R}_1^a) - \underline{T}^a, \tilde{L}_1) = U(w_1 \gamma(\underline{T}^a) F(Z - \tilde{L}_2, \bar{R} - \bar{R}_1^a), \tilde{L}_2). \quad (\text{A.38})$$

To show that there exists a sustainable and efficient allocation that satisfies Equations (A.34) – (A.38), we consider the extreme values of the utility functions achievable at  $t = 1a$ . Among all efficient allocations the maximal utility level for generation 1 is achieved by not investing anything in technical progress  $\underline{T}^a = 0$  and giving it all of the resource results in  $U_1^{max} = U(w_1 F(Z - \tilde{L}_1, \bar{R}), \tilde{L}_1)$  and  $U_2^{min} = U(w_1 \gamma(0) F(Z - \tilde{L}_2, 0), \tilde{L}_2) = 0$ . The reverse is the efficient policy  $(\bar{R}_1^{min}, \bar{R}_2^{max}, \underline{T}^{max})$  that makes the second generation as well-off as possible. For this all the production of generation 1 must be invested as otherwise one could increase  $U_2$  even further with more investment. So we have maximal redistribution which leads to  $U_1^{min} = U(w_1 F(Z - \tilde{L}_1, \bar{R}_1^{min}) - \underline{T}^{max}, \tilde{L}_1) = 0$  where  $w_1 F(Z - \tilde{L}_1, \bar{R}_1^{min}) = \underline{T}^{max}$  and  $U_2^{max} = U(w_1 \gamma(\underline{T}^{max}) F(Z - \tilde{L}_2, \bar{R} - \bar{R}_1^{min}), \tilde{L}_2)$ . Both investment  $\underline{T}$  and smaller resource extraction  $\bar{R}_1, \bar{R}_2$  increase production of gen-

eration 2 - once via increased productivity  $\gamma'(T)$  and once via the production function  $\partial F/\partial R > 0$ . Any efficient policy  $(\bar{R}_1^a, \bar{R}_1^a, \underline{T}^a)$  increases production and therefore later consumption. As the utility function is concave in consumption an increase in  $U_2$  monotonically decreases  $U_1$ . Hence, as  $U_1^{max} > U_2^{min} = 0$  and  $U_2^{max} > U_2^{min} = 0$  and there is a monotonic relationship between  $U_2$  and  $U_1$  there exists a unique policy mix  $(\underline{T}^a, \bar{R}_1^a, \bar{R}_2^a)$  that satisfies all Conditions (A.34)–(A.38).

### A.3 Proof of Proposition 1

At time  $t = 1b$ , resource use  $R_1$  is already irreversibly sunk in production and cannot be revised any more. As a consequence, the social planner's remaining policy variables at this stage are  $\bar{R}_2$  and  $\underline{T}$ . Also, generation 1 has already spent its labor time in production and, therefore cannot increase its leisure level,  $L_1$ , any more. It can, however, still decide over how much of the intermediate good produced,  $Y_1$ , to consume and how much to invest.

(i) The allocation  $(\tilde{C}_1, \tilde{R}_1, \tilde{L}_1, \tilde{T}, \tilde{C}_2, \tilde{L}_2, \tilde{R}_2)$  is Pareto-efficient (Lemma 2). Although  $\bar{R}_2$  is still a policy variable, it can only be reduced as  $\bar{R}_1^a + \bar{R}_2^a = \bar{R}$  which would make the second generation only worse-off and be inefficient according to Condition (11). Therefore, resource use remains on the original levels  $\bar{R}_1^a = \bar{R}_1^b, \bar{R}_2^a = \bar{R}_2^b$ . For the allocation to be sustainable in  $t = 1b$  a higher investment  $\underline{T}^b > \underline{T}^a$  would be necessary to counter the damages  $D(\bar{R}_1^a)$ .

$$U(w_1 F(Z - \tilde{L}_1, \bar{R}_1^a) - \underline{T}^a, \tilde{L}_1) > U(w_1 \gamma(\underline{T}^a) F(Z - \tilde{L}_2, \bar{R}_2^a) - D(\bar{R}_1^a), \tilde{L}_2) \quad (\text{A.39})$$

However, as  $\partial U/\partial C > 0$  and  $\partial^2 U/\partial C^2 < 0$  any investment  $\underline{T}^b \neq \underline{T}^a$  induces a change

in the  $MRS_{L_1, C_1}$  which in turn prevents the attainment of Condition (7) for Pareto-efficiency:

$$\frac{\partial U(w_1 F(Z - \tilde{L}_1, \overline{R}_1^a) - \underline{T}^b, \tilde{L}_1)}{\partial L_1} / \frac{\partial U(w_1 F(Z - \tilde{L}_1, \overline{R}_1^a) - \underline{T}^b, \tilde{L}_1)}{\partial C_1} \neq -w_1 \frac{\partial F(Z - \tilde{L}_1, \overline{R}_1^a)}{\partial L_1} \quad (\text{A.40})$$

Therefore, there exists no feasible policy  $(\overline{R}_1^b, \overline{R}_2^b, \underline{T}^b)$  that is both efficient and sustainable.

(ii) Due to the damages  $D(\overline{R}_1^a)$  the social planner needs to resort to higher investment  $\underline{T}^{bs} > \underline{T}^a$  in technical progress to achieve sustainability. Under this policy the second generation optimizes:

$$\begin{aligned} \max_{C_2, L_2, R_2} U(C_2, L_2) \quad \text{s.t.} \quad & Y_2 = w_1 \gamma(\underline{T}^{bs}) F(Z - L_2, \overline{R}_2^{bs}) - D(\overline{R}_1^{bs}), \\ & C_2 = Y_2, R_2 \leq \overline{R}_2^{bs}, \\ & 0 \leq L_2 \leq Z \end{aligned} \quad (\text{A.41})$$

which yields  $(\hat{C}_2, \hat{L}_2, \hat{R}_2)$  as solution. As resource extraction carries no costs it will use the resource completely  $\hat{R}_2 = \overline{R}_2^{bs}$ . As the total endowment of the resource is too small to lead to corner solutions for the leisure level we have  $Z > L_2$ . This results in the following:

$$\max_{L_2} U(w_1 \gamma(\underline{T}^{bs}) F(Z - L_2, \overline{R}_2^{bs}) - D(\overline{R}_1^{bs}), L_2) \quad (\text{A.42})$$

The FOCs are:

$$\frac{\partial U}{\partial L_2} = \frac{\partial U}{\partial C_2} w_1 \gamma(\underline{T}^{bs}) \frac{\partial F(Z - \hat{L}_2, \overline{R}_2^{bs})}{\partial L_2} + \frac{\partial U}{\partial L_2} = 0 \quad (\text{A.43})$$

Solving Equation (A.43) for the MRS the following condition is derived:

$$\frac{\partial U(\hat{L}_2, \hat{C}_2) / \partial L}{\partial U(\hat{L}_2, \hat{C}_2) / \partial C} = -w_1 \gamma(\underline{T}^{bs}) \frac{\partial F(Z - \hat{L}_2, \overline{R}_2^{bs})}{\partial L_2} \quad (\text{A.44})$$

With this condition for the behavior of the second generation and the irreversible production decision  $\hat{Y}_1 = \tilde{Y}_1, \hat{L}_1 = \tilde{L}_1, \hat{R}_1 = \tilde{R}_1 = \bar{R}_1^a = \bar{R}_1^{bs}$  effect of the level of investment  $\underline{T}$  on utility can be derived:

$$\text{for Generation 1} \quad U(w_1 F(Z - \hat{L}_1, \bar{R}_1^{bs}) - \underline{T}, \hat{L}_1), \quad (\text{A.45})$$

$$\text{for Generation 2} \quad U(w_1 \gamma(\underline{T}) F(Z - \hat{L}_2, \bar{R} - \bar{R}_1^{bs}) - D(\bar{R}_1^{bs}), \hat{L}_2) \quad (\text{A.46})$$

As the utility function is concave in  $C$  an increase of  $\underline{T}$  monotonically decreases the first generation's utility level in Equation (A.45) while increasing technical progress  $\gamma'(\underline{T}) > 0$  monotonically increases consumption  $C_2$  and therefore monotonically increases the second generation's utility level in Equation in Equation (A.46).

The range the transfer can take is  $0 \leq \underline{T} \leq \hat{Y}_1$ . The utility level of Generation 1 for  $\underline{T} = 0$  is  $U_1^{max} = U(w_1 F(Z - L_1, \bar{R}_1^{bs}), L_1)$  which is bigger than utility for Generation 2  $U_2^{min} = U(w_1 \gamma(0) F(Z - L_2, \bar{R}_2^{bs}), L_2)$  as  $1 < \gamma(0) < \gamma^s$ , i.e technical progress is no solution for the sustainability problem. For  $\underline{T} = \hat{Y}_1$  we have:  $U_1^{min} = U(0, \hat{L}_1)$  which is less than  $U_2^{max} = U(w_1 \gamma(\hat{Y}_1) F(Z - \hat{L}_2, \bar{R} - \bar{R}_1^{bs}) - D(\bar{R}_1^{bs}), \hat{L}_2)$  as  $U(0, L_t) = 0$  and the damages  $D(\bar{R}_1^{bs})$  can be offset by the investment  $\hat{Y}_1$ .

Therefore, there exists a  $\underline{T}^{bs}$  for which:

$$U(w_1 F(Z - \hat{L}_1, \bar{R}_1^{bs}) - \underline{T}^{bs}, \hat{L}_1) = U(w_1 \gamma(\underline{T}^{bs}) F(Z - \hat{L}_2, \bar{R} - \bar{R}_1^{bs}) - D(\bar{R}_1^{bs}), \hat{L}_2) \quad (\text{A.47})$$

As shown in appendix A.3 for  $\underline{T}^{bs} > \underline{T}^a$ , Condition (7) is not met. From Equation (A.47) it follows that the allocation is sustainable for the higher investment in technical progress.

Therefore, the sustainability policy  $\bar{R}_1^{bs}, \bar{R}_2^{bs}, \underline{T}^{bs}$  yields an allocation  $\hat{C}_1 = \tilde{Y}_1 -$



$\underline{T}^{bs}, \hat{L}_1 = \tilde{L}_1, \hat{R}_1 = \bar{R}_1^a = \bar{R}_1^{bs}, \underline{T}^{bs}, \hat{C}_2 = \hat{Y}_2, \hat{L}_2, \hat{R}_2 = \bar{R}_2^a = \bar{R}^{bs}$  that is not Pareto-efficient, but sustainable.

(iii) If the efficient allocation  $(\tilde{C}_1, \tilde{L}_1, \tilde{R}_1, \tilde{T}, \tilde{C}_2, \tilde{L}_2, \tilde{R}_2)$  from appendix A.2 is not changed at time  $t = 1b$  by enforcing investment in technical progress above the efficient level  $\underline{T}^{be} = \underline{T}^a$  the allocation remains efficient. However, the damages  $D(\bar{R}_1^a)$  make the resulting allocation unsustainable as in Equation (A.39). So the efficient policy  $(\bar{R}_1^{be}, \bar{R}_2^{be}, \underline{T}^{be})$  yields an allocation  $(C_1^*, L_1^*, R_1^*, T^*, C_2^*, L_2^*, R_2^*)$  that is efficient, but not sustainable. It equals the original allocation  $(\tilde{C}_1, \tilde{L}_1, \tilde{R}_1, \tilde{T}, \tilde{C}_2, \tilde{L}_2, \tilde{R}_2)$  with the damages  $D(\bar{R}_1^a)$ .

## A.4 Proof of Proposition 2

Just as in Appendix A.2 one can show that there exists an efficient and sustainable allocation. Note that at this stage resource extraction of generation 1 has already happened and cannot be regulated in retrospect. The damages are therefore not part of this consideration of efficiency. Again, we consider the extreme values of the utility functions achievable at  $t = 1b$ . Among all efficient allocations the maximal utility level for generation 1 is achieved by not investing anything in technical progress  $\underline{T}^b = 0$  and giving it all of the resource results in  $U_1^{max} = U(w_1 F(Z - \tilde{L}_1, \bar{R}), \tilde{L}_1)$  and  $U_2^{min} = U(w_1 \gamma(0) F(Z - \tilde{L}_2, 0) - D(\bar{R}), \tilde{L}_2) = 0$ . The reverse is the efficient policy  $(\bar{R}_1^{minb}, \bar{R}_2^{maxb}, \underline{T}^{maxb})$  that makes the second generation as well-off as possible. For this all the production of generation 1 must be invested as otherwise one could increase  $U_2$  even further with more investment. So we have maximal redistribution which leads to

$U_1^{minb} = U(w_1 F(Z - \tilde{L}_1, \overline{R}_1^{minb}) - \underline{T}^{maxb}, \tilde{L}_1) = 0$  where  $w_1 F(Z - \tilde{L}_1, \overline{R}_1^{minb}) = \underline{T}^{maxb}$   
 and  $U_2^{maxb} = U(w_1 \gamma(\underline{T}^{maxb}) F(Z - \tilde{L}_2, \overline{R} - \overline{R}_1^{minb}) - D(\overline{R}_1^{minb}), \tilde{L}_2)$ . Both investment  
 $\underline{T}^b$  and smaller resource extraction  $\overline{R}_1, \overline{R}_2$  increase production of generation 2 - once via  
 increased productivity  $\gamma'(\underline{T})$  and once via the production function  $\partial F / \partial R > 0$ . Any  
 efficient policy  $(\overline{R}_1^b, \overline{R}_2^b, \underline{T}^b)$  increases production and therefore later consumption. As  
 the utility function is concave in consumption an increase in  $U_2$  monotonically decreases  
 $U_1$ . Hence, as  $U_1^{maxb} > U_2^{minb} = 0$  and  $U_2^{maxb} > U_2^{minb} = 0$  and there is a monotonic re-  
 lationship between  $U_2$  and  $U_1$  there exists a unique policy mix  $(\underline{T}^b, \overline{R}_1^b, \overline{R}_2^b)$  that satisfies  
 all Conditions (A.21)–(A.27) for Pareto-efficiency.

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