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An Anti-windup Algorithm by Using Orthonormal Haar Functions in Wavelet Packets

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1 Abstract

This paper proposes an adaptive anti-windup (AW) compensation scheme for linear control systems with saturating actuators. A heuristic online control law is proposed using library of Haar wavelet packets in order to build a compensator controlling the reset windup phenomenon. An example is included to illustrate the effectiveness of the proposed method.

2 Introduction and Motivation

Industrial processes impose nonlinear limits on their process variables, while linear design techniques assume that there are no such limits. All physical systems need actuators for achieving control and these are subject to saturation. When this happens the feedback loop is effectively broken and if a regulator with an integrator is used the error will continue to be integrated. The value at the output of the regulator can become very large and often degrades the closed-loop performance in the form of large overshoot, long settling time and sometimes even instability. This is more evident if compared with the expected linear performance for the systems. The phenomenon described is called "windup". The windup phenomenon has attracted interest in academic and in industrial community, already at the end of the eighties, see [1], the problem became to have the first solution, an overview of the basic schemes is available in [5]. More recent work like in [4] analyses the conditions in order to find invariant subspaces and in [6] the preaction is employed to enhance the performance. Wavelets and wavelet packets are becoming popular at the beginning of the nineties, see for instance [3], and their applications are already developed in many and different field. Recent works in wavelet [9, 10] have indicated wavelets as a promising approach for off line analysis, monitoring and classification. More, recent works [7, 11] developed efficient algorithms in order to detect and classify transient harmonic phenomena as pantograph vibration and *inrush current* respectively. Wavelets have shown to be properly suitable to analyze electrical transients and noise. Among the wavelet family we have chosen the Haar basis, which is particularly suitable for the analysis of square-waveform functions.

This paper tries to apply wavelet tool in an anti-windup scheme in order to use time frequency information of the windup phenomenon. A heuristic adaptive control law is calculated using Haar bases in wavelet packets. The least squares method is applied to calculate a reset windup function optimizing the energy index error as in [8] but independently from the knowledge of the plant parameters. The energy index is optimized over subspaces corresponding to the time frequency cells where the saturating input signals and

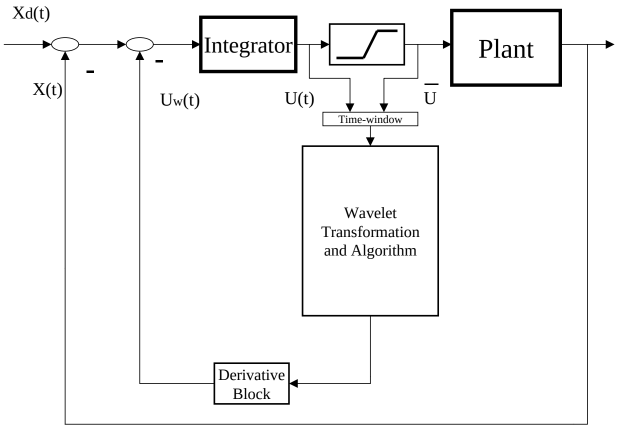

Figure 1: Anti-windup scheme using wavelet.

the desired input signals have the biggest distance. In other words the signal belonging to the *saturating subspaces* is calculated and used to reset the windup. The Haar functions fit quite good the saturating input signals and show good precision and thus a quick switch-off. The particular structure of the wavelet tree allows to construct efficient and effective algorithms, known and consolidated DSP techniques can implement them. It is known, in fact, that for a time signal of length N (where N must be a power of two) the Fast Fourier Transform (FFT) requires order $N \log_2 N$ operations, while the Wavelet Transform needs only order N operations, which is the best possible.

The idea is very simple and it is the basis for every problem formulation of the anti-windup [5, 12]: it is necessary to recognize *where and when* that windup is present and to find a feedback compensator for resetting to integrator to a 'normal' value.

3 Problem Statement

Without loosing generality we will consider the basic scheme as represented in Fig. 1. Let $\tilde{x}_c(t) = x_c(t) - \bar{x}_c(t)$ be the mismatch between the constrained and unconstrained closed-loop scheme for the integral compensator as represented in Fig. 1.
One considers the two different compensator dynamics:

if $\tilde{x}_c(t) > \delta$, where δ is the saturating level of the actuators, then

$$\begin{aligned} \dot{x}_c(t) &= x_d(t) - x(t) + u_w(t) \\ u_c(t) &= x_c(t) \end{aligned} \tag{1}$$

if $\tilde{x}_c(t) \leq \delta$ then

$$\begin{aligned} \dot{x}_{dc}(t) &= x_d(t) - x(t) \\ u_{dc}(t) &= x_{dc}(t) \end{aligned} \tag{2}$$

The classical anti-windup schemes normally developed for PI or PID controllers, involve turning off the integral action in the presence of saturation. In the literature, windup is interpreted as an inconsistency of the output or the state of the regulator between the case with and the case without saturation (see [12] and [8] respectively).

According to the Fig. 1 we can define two cost functions:

$$\mathcal{J}_1 = \int_0^\infty \|u(t) - \bar{u}(t)\|^2 d(t) \tag{3}$$

and

$$\mathcal{J}_2 = \int_0^\infty \|x_c(t) - \bar{x}_c(t)\|^2 d(t) \tag{4}$$

where $\| \cdot \|$ is the Euclidean norm, with $\bar{u}(t)$ and $\bar{x}_c(t)$ we have indicate respectively the output and the state of the regulator in case of saturation. In our case the two indices are equivalent.

The problem can be formulated in the following way.

Given the scheme depicted in Fig. 1 which can be efficiently represented for the compensator by (1) and by (2). Find a controlling law $u_w(t)$ such that the index defined in (4) is optimized.

One will see how, using the Haar wavelet tool, it results relatively easy to find the control law.

Given now a function $f(t)$ then loosely speaking, the function $f(t)$ can be decomposed into

$$f(t) = \sum_j \sum_n w_{(d,j,n)} \psi_{(d,j,n)}(t) \tag{5}$$

where the $\psi_{(d,j,n)}(t)$ are the wavelet functions, normally obtained by dilating and translating a mother function $\psi(t)$, the index j and n denote the dilation and translation respectively and d is the level of the tree, $w_{(d,j,n)}$ is the weight coefficient for $\psi_{(d,j,n)}(t)$.

The proposed method is related to the *Parseval's Theorem* and *Time-Frequency Content* for a signal. If we are using orthonormal bases as in (5) then the Parseval's Theorem relates the energy in each of the expansion components and their wavelet coefficients. Given a tree of the orthonormal wavelet bases then the signal $f(t)$ can be represented as follows:

$$\int |f(t)|^2 d(t) = \sum_j \sum_n |w_{(d,j,n)}|^2. \tag{6}$$

The energy of the signal in time domain is partitioned at different resolution levels.

The problem statement can be summarized as follows:

- *Choose previously the best family which represents in the best way the signals.*

- *Choose the best (d, j, n) cells where to describe the wanted signal.*

- *Calculate the control law $u_w(t)$ which optimizes, in a heuristic way, the index defined in (4).*

4 Background

We describe very quickly an orthogonal wavelet basis of $\mathcal{L}^2(\Re)$. The oldest example of wavelet functions are the Haar functions, $\psi^h_{(d,j,n)'}(t)$, derived from the Haar mother wavelet:

$$\psi^h_{(1,0,0)}(t) = \begin{cases} 1 & 0 \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} \leq t < 1 \\ 0 & otherwise. \end{cases}$$

Where 'd' is a scale parameter, 'j' is a phase parameter and 'n' is a time translation parameter. In order to understand better we will show an example of a set of Haar wavelet function which represents a part of a 'Haar wavelet packet tree', see Fig. 3, one will see later a bit more in depth.

Basically the Haar basis has the two following property:

- the $\psi^h_{(d,j,n)}(t)$ are orthonormal;

- any $\mathcal{L}^2(\Re)$ function f can be approximated, up to arbitrarily small precision, by a finite linear combination of the $\psi^h_{(d,j,n)}(t)$.

In particular, to be more precise, the coefficients $w_{(d,j,n)}$ (the weight coefficients) are calculated as follows:

$$w_{(d,j,n)} = \int_I f(t)\psi^h_{(d,j,n)}(t)d(t).$$

Where $f(t)$ is the wanted signal and I is the considered time interval.

More in depth, to each wavelet packet function one can associate a time $\mathbf{t}$ and a frequency $\mathbf{f}$. These will be uncertain by amounts $\mathbf{\Delta t}$ and $\mathbf{\Delta f}$, respectively. The result may be interpreted as a rectangular patch of dimensions $\mathbf{\Delta t}$ by $\mathbf{\Delta f}$, located around $(\mathbf{t}, \mathbf{f})$. One will call the patch a phase cell, or Heisenberg box, in honor of the uncertainty principle, which limits how small the area of the patch may be. An orthonormal basis corresponds to a disjoint cover of the phase plane by phase cells (Heisemberg boxes). One representation of this phase aspect is depicted in Fig. 2.

More rigorously,

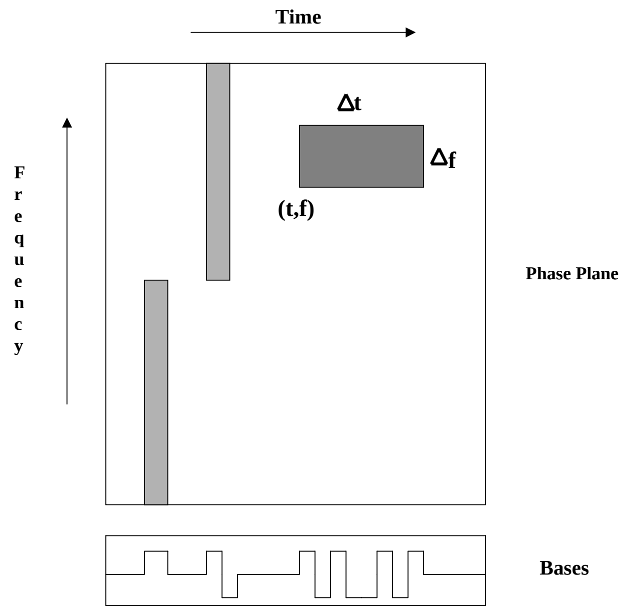

Figure 2: Time frequency cells in phase plane.

Definition 1 *Let a tree of wavelet packets be the collection of functions of the form*

$$\psi_{(d,j,n)}(t) = \psi_j(2^d t - n) \qquad (7)$$

where $(d, n) \in \mathsf{Z}$ *and* $j \in \mathsf{N}$.

We are talking about truncated indices, thus finite libraries of wavelet packets. Here, the *pyramidal* packet is represented with the indices (d, j, n), d is the level of the tree (scaling parameter), j is the frequency cell (oscillation parameter) and n the time cell (localization parameter).

The function $\psi_{(d,j,n)}(t) = \psi_j(2^d t - n)$ has support of size 2^{-d} of the Nyquist frequency.

To go a little bit more in depth, we suppose that the signal consists of $N = 2^{N_0}$ dyadic and equally spaced samples and the library tree contains all the Haar function analyses to level N_0 of the frame, with windows of size $2^{N_0}, 2^{N_0-1}, ...1$. The basis function will be indexed by the triplet (d, j, n): if N is the total number of the samples then the corresponding samples related to the d *level* with relative desampling are $N_d = 2^d$ and $0 \leq d \leq N_0$, $0 \leq n < 2^{N_0-d}$, $0 \leq j < 2^d$-1.

The scale parameter d gives the number of decompositions of the original signal window into subwindows and the position index n numbers the adjacent windows. *Thus the information cell is drawn over the vertical axis (frequency) interval* $I_j = [2^{N_0-d}j, 2^{N_0-d}(j+1)[$. In general, the subspace over the time subinterval I_n consists of the function

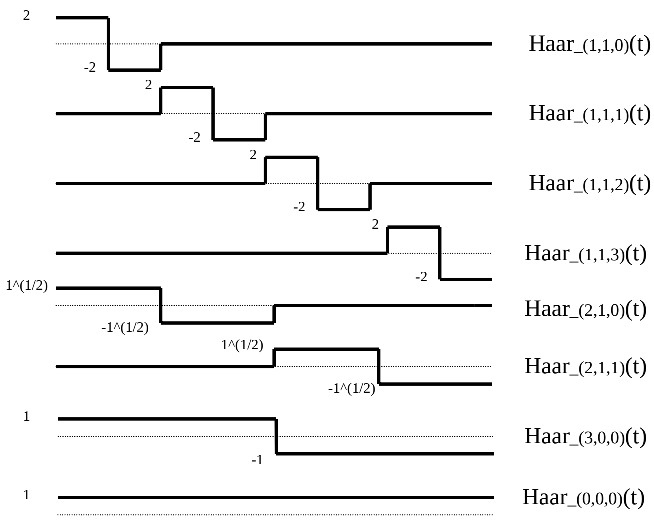

Figure 3: A set of Haar functions.

with the associated information cell alongside the time interval $I_n = [2^d n, 2^d(n+1)[$ on the horizontal (time) interval. Each subdivision halves the nominal window width and thus the resolution level, in particular the resolution level on the tree could be represented like a collection of rectangles:

$$[2^{N_0-d}j, 2^{N_0-d}(j+1)[\times[2^d n, 2^d(n+1)[. \qquad (8)$$

4.1 A Spectral Distance

We need a measure of the distance between two signals in order to find the time-frequency cells where the saturating input signals and the desired input signals have the big mismatch. As there exist many spectral distance measures, every measure dedicated to one particular application, for an overview see [2], we define the distance between two signals in wavelet domain in the following way:

Definition 2 *Given two signals $s_0(t)$ and $s_1(t)$ and their wavelet decomposition:*

$$s_0(t) = \sum_j \sum_n w_{0_{(d,j,n)}} \psi_{(d,j,n)}(t)$$

and

$$s_1(t) = \sum_j \sum_n w_{1_{(d,j,n)}} \psi_{(d,j,n)}(t).$$

One defines distance $\mathcal{D}_{(d,j,n)}(s_0, s_1)$ between $s_0(t)$ and $s_1(t)$ $\forall\ (d,j,n)$ the function

$$\mathcal{D}_{(d,j,n)}(s_0, s_1) = |||\frac{w_{1_{(d,j,n)}}}{w_{0_{(d,j,n)}}}| \log |\frac{w_{1_{(d,j,n)}}}{w_{0_{(d,j,n)}}}|||. \qquad (9)$$

Where the indices (d,j,n) indicates the time-frequency cells in the wavelet packet decomposition.

Remark 1 *One wants to remark that this distance is not a true distance, for instance it doesn't satisfy the symmetric property. One doesn't need to define a metric space but one needs just to define a distance which can illuminate the difference between two signals.*

Remark 2 *It is easy to see that the distance $\mathcal{D}_{(d,j,n)}(s_0, s_1)$ defined in (12) is stronger*[1] *than the spectral distance defined in wavelet domain $\mathcal{D}_{s_{(d,j,n)}}(s_0, s_1) = \| \log |\frac{w_{1_{(d,j,n)}}}{w_{0_{(d,j,n)}}}|||$ $\forall\ (j,n)$ $|w_{1_{(d,j,n)}}| > |w_{0_{(d,j,n)}}|$.*

The latter remark indicates the measure defined in (12) like an efficient measure in order to pick up a suitable subspace where to represent the signals. More, the above defined index is very quick to calculate. The proposed algorithm will try to find an input function $u_w(t)$ which optimizes the index defined in (4) subtracting the signal belonging to the subspaces where the distance between the saturating input and the not saturating input has maximum.

5 The proposed Algorithm

Given a suitable dyadic time window, let the index i the sampling index and

$$\mathcal{H} = \left\{\psi^h_{(d,j,n)}(t); (d,n) \in \mathsf{Z}, j \in \mathsf{N}, t \in \Re\right\}$$

the truncated Haar wavelet packet tree which we will consider. If we think the signal in the above mentioned time window then the algorithm can be represented as follows:

Begin Loop
for $i = 1$ to ∞

$\forall\ \bar{t}$ **such that** $|x_c(\bar{t})| \leq \bar{x}_c$ **then** $u_w(t) = 0$
else

Let

$$f_{1_i}(t) = \int_0^t u_{w_i}(\tau) d\tau$$

$$f_{2_i}(t) = \delta - \int_0^t \big(x_d(\tau) - x(\tau)\big) d\tau.$$

[1] One recalls that a distance $\mathcal{D}_1$ is said *stronger* than the distance $\mathcal{D}_2$ if $\forall\ (s_0, s_1)$ $\mathcal{D}_1(s_0, s_1) \geq \mathcal{D}_2(s_0, s_1)$.

step 1

Decompose on all the Harr packet tree the signal $f_{2_i}(t)$:

$$\mathcal{J}\Big(\mathbf{h}_{(d,j,n)}\Big) =$$

$$\Big(f_{2_i}(t) - \sum_{(d,j,n)\in\mathcal{H}} \mathbf{h}_{(d,j,n)}\psi^h_{(d,j,n)}(t)\Big)^2 \qquad (10)$$

this yields:

$$\mathbf{h}_{(d,j,n)} = \Big(\frac{\sum_{(d,j,n)\in\mathcal{H}}\langle f_{2_i}(t), \psi^h_{(d,j,n)}(t)\rangle}{\Big(\sum_{(d,j,n)\in\mathcal{H}}\big(\psi^h_{(d,j,n)}(t)\big)^2\Big)^{(-1)}}\Big). \qquad (11)$$

step 2

Select the subspaces such that:

$$\{l_{h_{(d,j,n)}}\} = \mathsf{arg}\Big(\max_{(d,j,n)} \mathcal{D}_{(d,j,n)}(x_c(t), \bar{x}_c(t))\Big) =$$

$$\max_{(d,j,n)}\big(\|| \frac{w_{c_{(d,j,n)}}}{\bar{w}_{c_{(d,j,n)}}}|\log|\frac{w_{c_{(d,j,n)}}}{\bar{w}_{c_{(d,j,n)}}}|\||\big). \qquad (12)$$

step 3

$$f_{1_i}(t) = \sum_{l_h\in\mathcal{H}} \mathbf{h}_{l_{h_{(d,j,n)}}}\psi^h_{l_{h_{(d,j,n)}}}(t);$$

$$u_{w_i}(t) = \frac{d}{dt}\big(f_{1_i}(t)\big).$$

end loop

6 Simulation

In this section one example is shown to demonstrate the effectiveness of the proposed method depicted in Fig. 4. The matlab function which we have defined are coherently chosen according the description in paragraph 5. We have chosen a wavelet packet tree with four level, in particular the basis function $\psi^h(t)$ was chosen equal to Haar function in order to *illuminate* the difference between the saturating and the not saturating input control. In Fig. 7 two adjacent Haar functions corresponding to the fourth level are reported. The window which we are considering is equal to 350 ms, this is the dominant dynamic of the considered system. In Fig. 5 it is possible to see how the output is considerable improved. In fact the saturating inputs are switched-off in a short time-period as reported in Fig. 6.

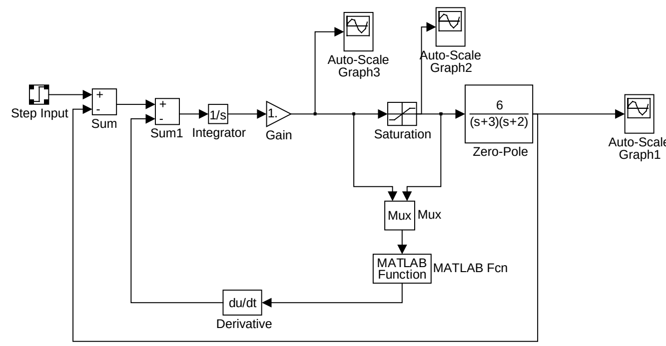

Figure 4: Simulink/Matlab simulation scheme.

6.1 Discussion

The idea behind the algorithm is to analyse the signal in input to the saturation through the wavelet tree at every sample. One needs a time window in order to backup the signal, the length of the window should be dimensioned according to the frequency resolution which one wants to achieve for the signal analysis, this in fact depend also on the Nyquist frequency[2]. In other words the idea is to decompose the saturating signal over the wavelet tree in order to be able to recognize the saturating part, from the not saturating part. This technique has several drawbacks, in fact if one chooses a too short window one doesn't have high frequency selectivity When the window is too long the frequency selectivity is very high, but one has information too localized in the past. This aspect could be also dangerous for the stability of the loop.

The empirical design of the window dimension and a possible weighted window could be possible solution. In case of disturbance in output of the system the empirical choice becomes a bit more difficult, several a priori knowledge on the noise is in this case becomes necessary.

7 Conclusion

A heuristic adaptive time frequency anti-windup compensation method is proposed. Haar wavelet bases are adopted in order to find a control law which guarantees good performances optimizing the energy error index. An online control law running through the Haar wavelet tree is proposed. The anti-windup compensator is not based on the model of the controlled plant, the robustness of the method is guaranteed.

[2]One recalls that $R = \frac{N_f * 2^{(-d)}}{N}$ where N_f is the Nyquist frequency (Half of the sampling frequency), 'd' is the level of the considered tree and N is the length of the signal.

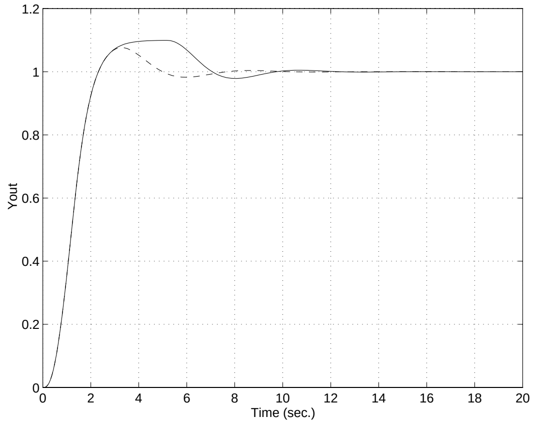

Figure 5: Output. Solid: Without Antiwindup control. Dashed: With Antiwindup Control.

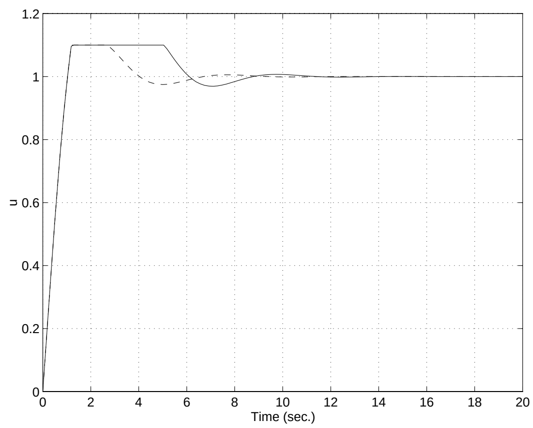

Figure 6: Saturating Input. Solid: Without Antiwindup control. Dashed: With Antiwindup Control.

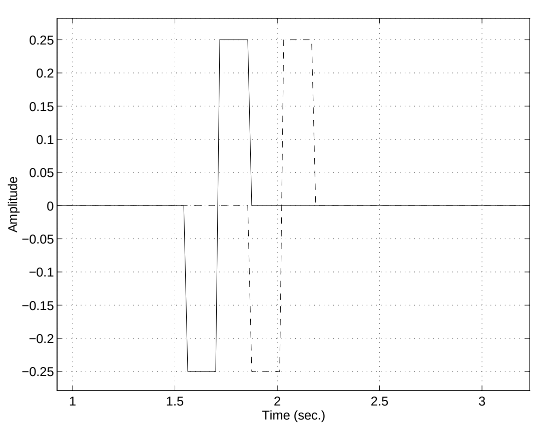

Figure 7: Detail of the orthonormal Haar functions.

References

[1] K. L. Aström and L. Rundqwist. Integrator windup and how to avoid it. In *Proc. of the ACC'87 American Control Conference*, Pittsburgh, 1987.

[2] M. Basseville. Detecting changes in signal and systems: A survey. *Automatica*, 24(3):309–326, 1988.

[3] I. Daubechies. *Ten Lectures on Wavelets*. Publisher Society for Industrial and Applied Mathematics, Philadelphia (Pennsylvania), 1995.

[4] N. Kapoor, A. R. Teel, and P. Daoutidis. An invariant subspace technique for anti-windup syntesis. In *Proc. of ACC'97 American Control Conference*, Albuquerque New Messico (U.S.A.), 1997.

[5] M. V. Kothare, P. J. Campo, M. Morari, and C. N. Nett. An unified framework for the study of anti-windup designs. *Automatica*, pages 1869–1883, 1994.

[6] P. Mercorelli, B. Paden, and D. Prattichizzo. Perfect anti-windup in output tracking scheme with preaction. In *Proc. of CCA'98 IEEE International Conference on Control Applications*, Trieste (Italy), 1998.

[7] P. Mercorelli, M. Rode, and P. Terwiesch. A wavelet packet algorithm for online detection of pantograph vibrations. In *Proc. 9 th IFAC Int. Symp. on Control in System Transportation 2000, in press*, Braunschweig, 2000.

[8] J. K. Park and C. H. Choi. Dynamic compensation method for multivariable control systems with saturating actuator. *IEEE Transaction on Automatic Control*, pages 1635–1640, 1995.

[9] B. Perunicić, M. Mallini, Z. Wang, and Y. Liu. Power quality disturbance detection and classification using wavelets and artificial neural networks. In *Proc. 8th IEEE International Conference on Harmonics and Quality of Power*, Athens 1998.

[10] P. Terwiesch, S. Menth, and S. Schmidt. Analysis of transients in electrical railway networks using wavelets. *IEEE Trans. on Industrial Electronics*, 45(6):955–959, 1998.

[11] P. Terwiesch and P. Mercorelli. A local feature extraction using biorthogonal bases for classification of embedded classes of signals. In *Proc. 14 th Int. Symp. on Mathematical theory of networks and systems (MTNS 2000), in press*, Perpignan (France), 2000.

[12] A. Zheng, M. V. Kothare, and M. Morari. Anti-windup design for internal model control. *INT. J Control*, (5):1015–1024, 1994.