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NONLINEAR DYNAMICS AND OPINION FORMATION IN TIME VARYING NETWORKS

Abstract: The communication structures within our society can be visualized as networks, that are dynamic and change over time due to various factors. Numerous mathematical models have been developed to simulate opinion dynamics. These models are predominantly agent-based, where an opinion-forming process occurs through interactions between individual agents. The interaction processes are based on an underlying network of agents. The *DeGroot (DG) model* [1] is the most well-known continuous opinion space model. According to this model, a person's opinion is derived from their previous opinion and the influence process. In addition to *DG-based models*, there are also *continuous opinion space models with bounded confidence* [2]. These are characterized by individuals ignoring ideas or opinions that are too far removed from their own. However, these models do not assume an underlying interaction network, but rather assume interactions between all individuals in the population. We extend the *DeGroot-Friedkin (DGF) model* [3] with constant self-weights for the development of social influence networks to a bounded confidence model. Based on this extended DGF model, we analyze opinion-forming processes in different network topologies.

The extension of the DeGroot-Friedkin model to a bounded confidence model

The DGF model uses a row-stochastic interaction matrix $C \in \mathbb{R}^{n\times n}$ to describe the mutual interactions of $n \in \mathbb{N}$ agents. The vector $y(t) \in \mathbb{R}^n$ describes the opinions of the agents at time $t \in \mathbb{Z}$, where $y_i(t) \in [-1,1]$. Each agent has a self-weight that indicates the relative control of keeping the own opinion, which is represented by the entries of vector $s \in \mathbb{R}^n$ with $s_i \in [0,1]$. Here we choose a constant s. The influence matrix $W(t)$ and the updated opinions are given by:

$$W(t) = \mathrm{diag}[s] + (I_n - \mathrm{diag}[s]) \cdot C \quad (1)$$

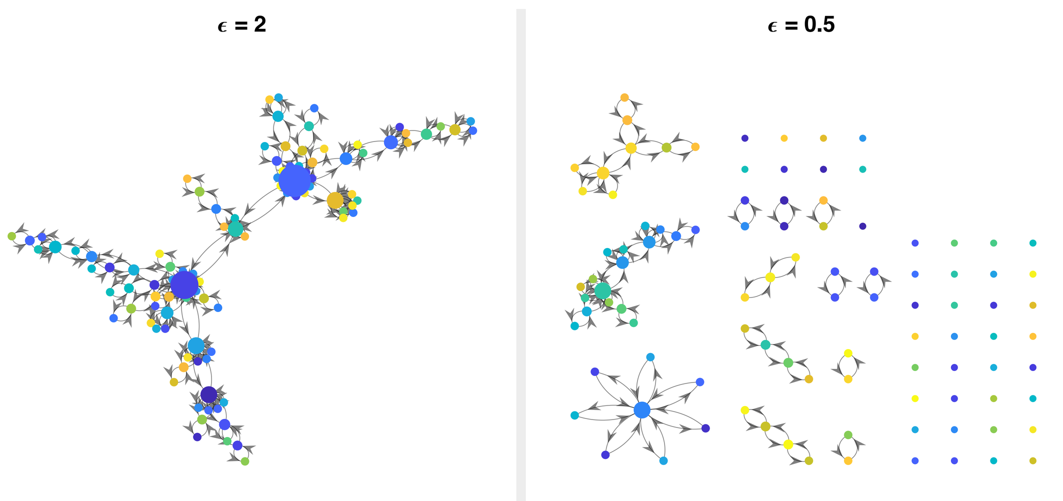
$$y(t+1) = W(t) \cdot y(t), \quad (2)$$

where $\mathrm{diag}[s]$ is the diagonal matrix with entries of s on its diagonal.

As an extension, an interaction partner's opinion now only affects the agent's own opinion if it falls within the predefined ϵ-neighborhood, where $\epsilon \in [0,2]$. For this, we substitute C in equation (1) by $C_\epsilon(t)$, which we obtain by deleting the corresponding entries in C, for which the agents' opinions are further apart than the chosen ϵ at t (and subsequent row-normalization).

Barabási-Albert (B-A) scale-free networks [4]

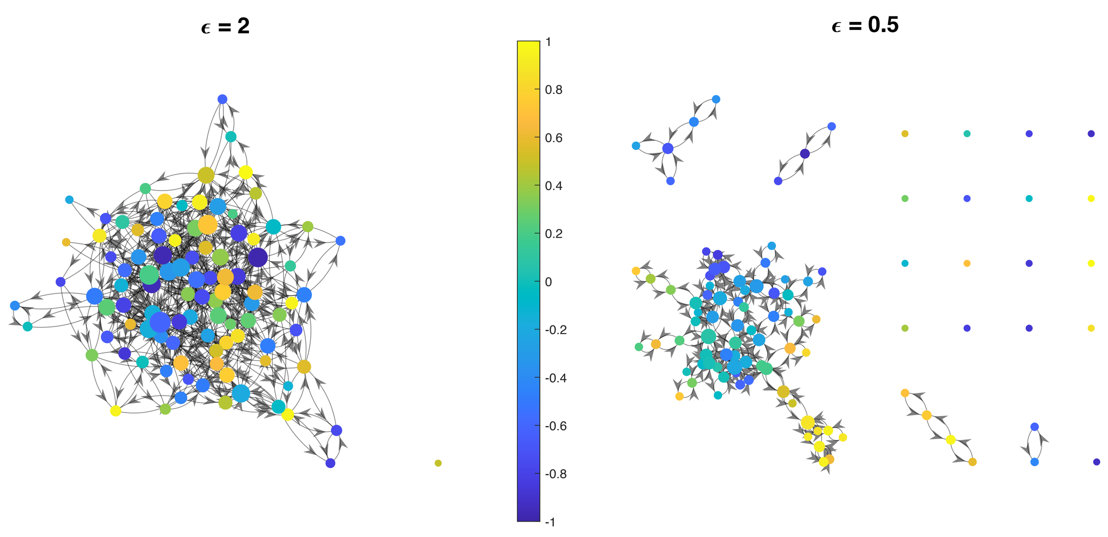
$n = 100, \quad m_0 = 5, \quad \mathrm{mlink} = 1$



The *B-A algorithm* starts with a small number of connected nodes, usually $m_0 = 5$ nodes. After each time step, another node is added to the network, with each new node being connected to $\mathrm{mlink} \leq m_0$ nodes already present in the network. A new node is connected to an existing node i with probability $\Pi(k_i) = \frac{k_i}{\sum_j k_j}$, where k_i is the degree of node i. After t time steps, the network has $n = t + m_0$ nodes and $\mathrm{mlink} \cdot t$ added edges.

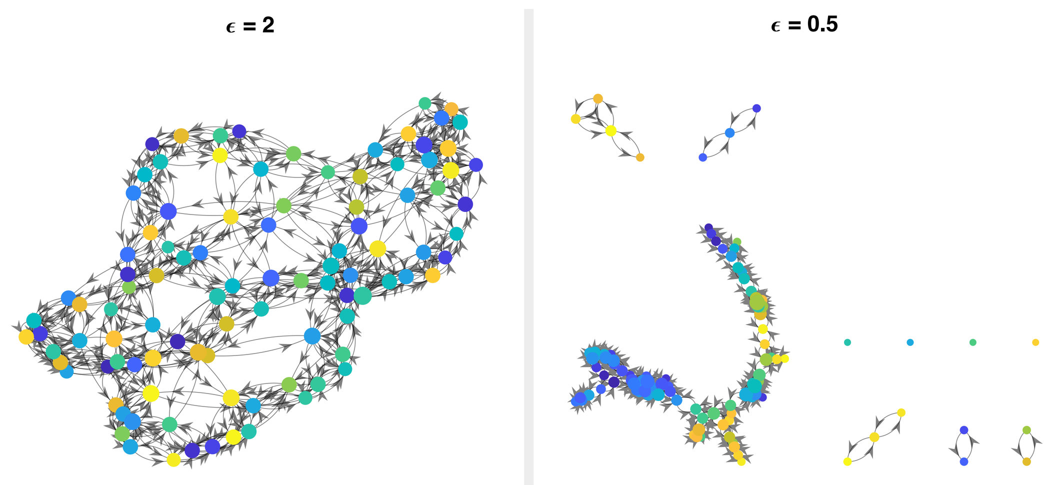
Random networks

$n = 100, \quad$ maximum possible interaction partners $= 5$



Watts-Strogatz (W-S) small-world networks [5]

$n = 100, \quad k = 3, \quad \beta = 0.05$

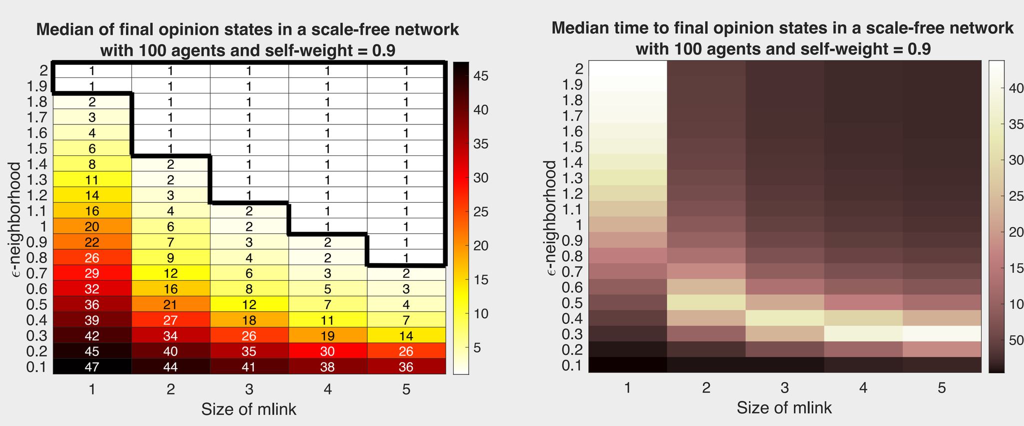


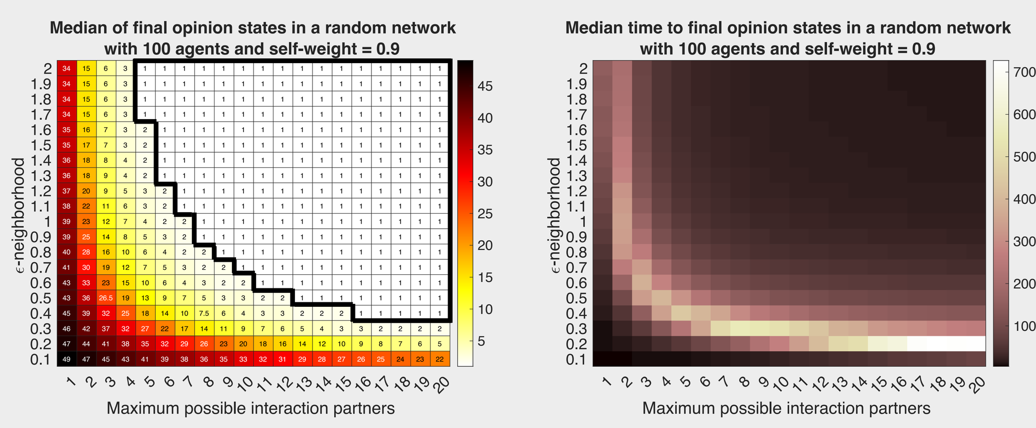
The *W-S algorithm* starts from a ring lattice with n vertices. Each vertex is connected to its $2 \cdot k$ nearest neighbors. Each edge is then rewired with probability β, where $0 \leq \beta \leq 1$. These networks can be highly clustered and still have small characteristic path lengths. Thus $2 \cdot k$ is still the average node degree. For $0.05 \leq \beta \leq 0.15$ the WS-algorithm generates a small-world network.

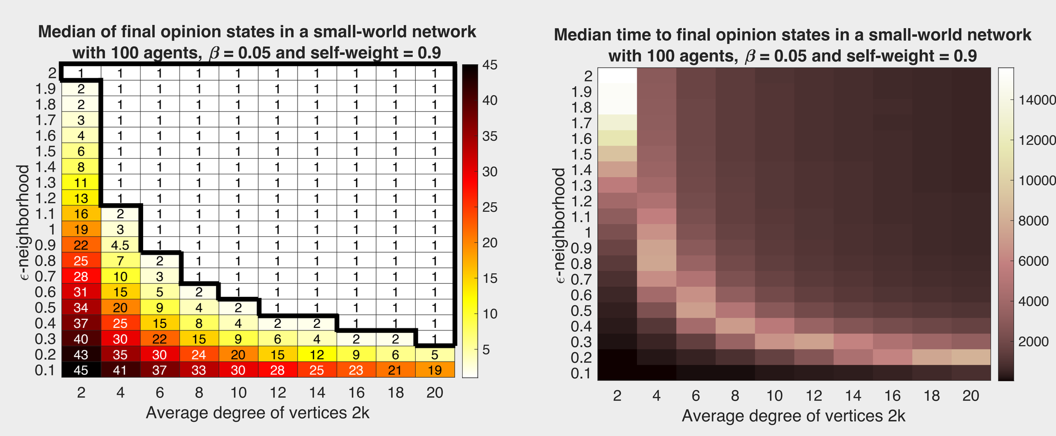
Results

In our simulations we use for each of a thousand runs a random interaction matrix based on a chosen network topology. The initial opinions $y_0 = y(0)$ of the n agents are generated randomly in each run.

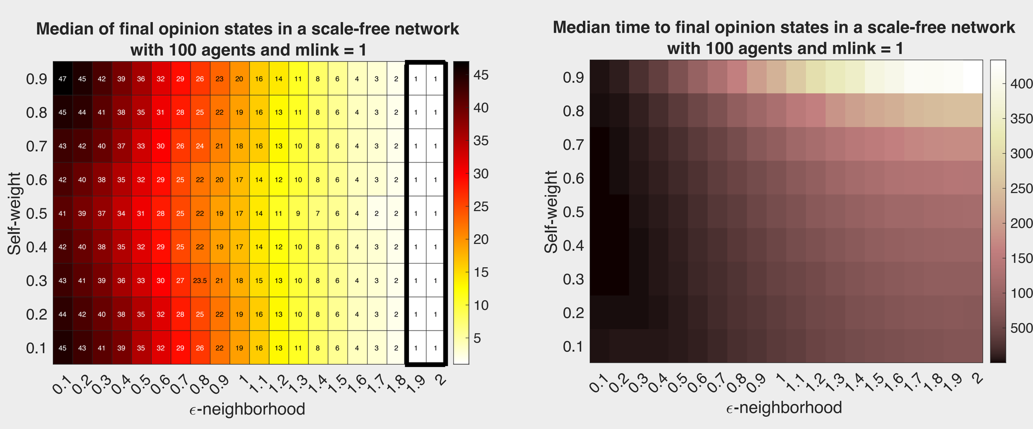
Impact of neighbors and ϵ

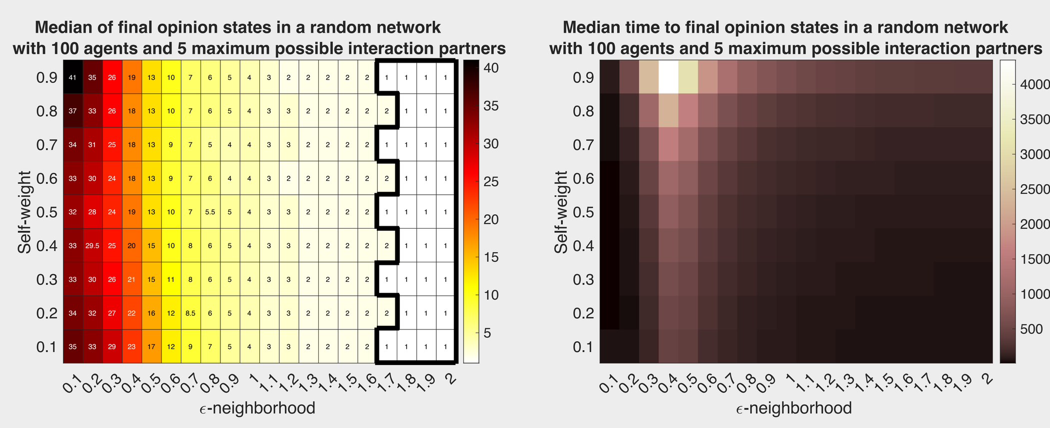


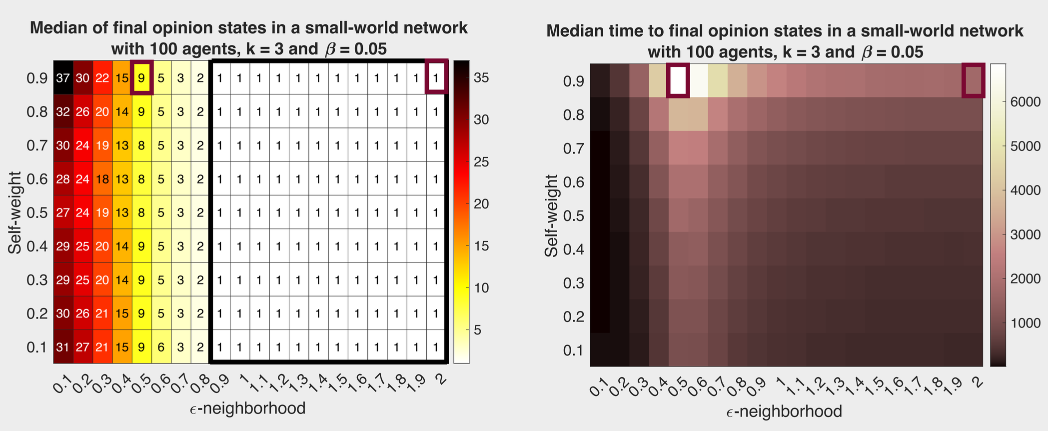




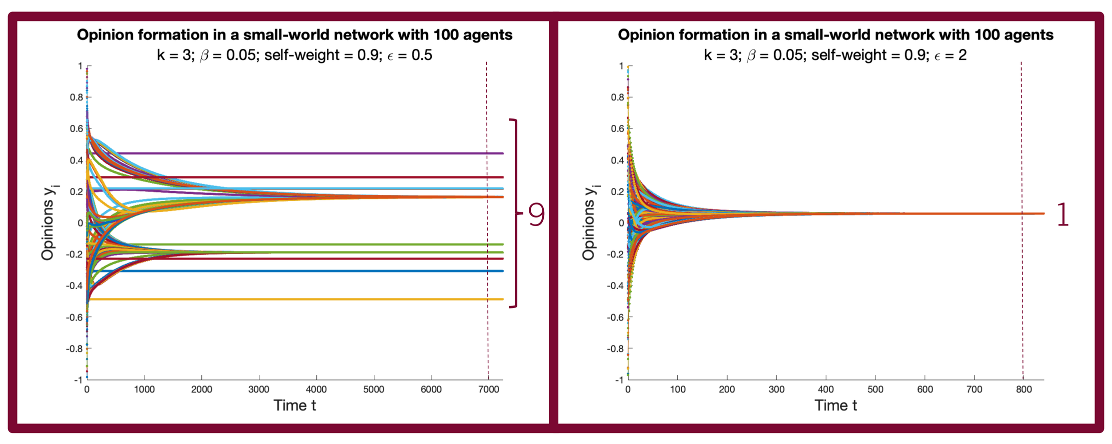
Impact of ϵ and self-weight







Outlook: Further, we introduce special agents that can influence the interaction partners outside of the ϵ-neighborhood. These special agents are therefore considered to represent social media because their opinions never change, and everyone is influenced by one of them. We simulate and analyze random opinion formations under different conditions, for example using random interaction matrices for weekdays and weekends, and examine the emergence of consensus, polarization, and coexistence of different opinions in different network topologies.



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