Resilience of natural-resource-dependent economies: weak vs. strong demand-side interactions

Abstract: We study the role of consumer needs for (limited) resilience of natural-resource-dependent economies. In particular, we study how substitutability vs. complementarity of natural resources in consumer needs may give rise for multiple steady states and path dependence under the optimally controlled harvest of two renewable natural resources. This is a major shift in the interpretation and analysis of resilience, from seeing (limited) resilience as an objective property of the economy-environment system to acknowledging its partially subjective, preference-based character. We show that the resilience of natural-resource-dependent economies decreases with the degree of complementarity between resources in consumer needs. More generally, we hypothesize that the stability of economic systems decreases with the strength of demand-side interactions.

Keywords: collapse, crisis, natural resources, resilience, tipping points

JEL-Classification: C62, O13, Q01, Q20

1. Introduction

Economies that use and depend on dynamic natural resources may be understood as dynamic systems exhibit non-trivial dynamics. This includes largely resilient stability domains, where exogenous shocks are effectively buffered, and, on the opposite, potential collapse of resource stocks and welfare. The former and the latter potential dynamics of one and the same system are separated by tipping points, i.e. threshold values of the economy-environment-systems state variables, which separate domains of fundamentally different dynamics. Long-term efficient and sustainable management of such systems is thus a huge challenge and requires a thorough understanding of the origins and mechanisms of such non-linear dynamics.

The collapse of historical natural-resource-dependent economies has become a fascinating area of economic research in recent years, which promises conclusions for todays major environmental crises such as climate change or biodiversity loss.[1] Examples include the societies of Easter Island, the Anasazi, and the Maya (Diamond 2005). Brander and Taylor (1998), Taylor (2009) and Good and Reuveny (2009) provide a modeling analysis of the collapse of the historical society of Easter Island, suggesting that the collapse may have been due to a nonlinear interaction between population growth and the dynamics of natural resource use. Such studies may provide important insights for today's decision making, as, on a global scale, economies depend on natural resources, such as clean water and the global climate system

[1] As a counter position, McAnany and Yoffee (2010) question the idea of collapse. They stress the large resilience of natural-resource-dependent societies.

(Arrow et al. 1995, Taylor 2009).

Traditionally, literature in resource economics has considered dynamics of resource use where the natural resource is either bound to depletion or can be harvested sustainably, depending on the growth dynamics of the resource, on harvesting technology and on the economy's institutions (Gordon 1954, Clark 1973, 1990, Copeland and Taylor 2009). More recent studies show that the dynamics of resource use may be path-dependent, such that the fate of the resource-dependent economy depends on its initial state. In terms of dynamical systems, this means that the system features more than one stable steady-state, with a domain of attraction for each. As a consequence, a system of resource use may flip from one domain of attraction into another one as a result of exogenous disturbance (Holling 1973, Levin et al. 1998, Carpenter et al. 2001, Scheffer et al. 2001). Such a limited resilience may be due to nonconvexities in the dynamics of the ecosystem, as for example, in boreal forests, semi-arid rangelands, wetlands, shallow lakes, coral reefs, or high-seas fisheries (Gunderson and Jr. 2002). The implications for the management of nonconvex ecosystems that have several stable states have been studied in general (Dasgupta and Mäler 2003), and in particular for shallow lakes (Mäler et al. 2003) and rangelands (Perrings and Stern 2000, Anderies et al. 2002, Janssen et al. 2004). Limited resilience may also be driven by harvesting technology or storability of resources in an open access setting (Kremer and Morcom 2000, Bulte 2003).

While previous studies have explained limited resilience by the dynamics of natural resources and technology, i.e. by the constraints of economic action, in this study we highlight the role of consumer preferences, i.e. on

the properties of the societal objective function. In particular, we study the role of substitutability vs. complementarity of natural resources in consumer preferences. This is a major shift in the interpretation and analysis of resilience, from seeing (limited) resilience as an objective property of the system to acknowledging its partially subjective, preference-based character.

For this sake, we develop and study a model with two renewable natural resources which enter consumer preferences with a constant elasticity of substitution that represents the full spectrum between perfect substitutes and perfect complements. Accordingly, the degree of complementarity between resources – which is the inverse of the elasticity of substitution in consumption – measures the strength of demand-side interactions between the two resources. The other element in the objective function that determines (limited) resilience is the time horizon of resource use, i.e. the societal discount rate. As most renewable natural resources are managed with some degree of cooperation (Ostrom 1990), we study resource use that is optimally planned rather than open-access resource use.

Our main results are conditions on the degree of complementarity of resources in consumer preferences and on the societal discount rate under which the dynamics of resource use features path dependency and limited resilience, i.e. exhibits multiple stable steady states. We show that the resilience of natural-resource-dependent economies decreases with the degree of complementarity between resources in consumer preferences.

The paper is organized as follows. In Section 2 we develop a stylized model of an economy that depends on the use of two renewable natural resources. Section 3 derives the conditions for the optimal management of these

resources. In Section 4 we study how the number and stability properties of optimal steady states depends on the degree of complementarity of the two resources and on the societal discount rate, providing both analytical results and numerical examples. Section 5 concludes and discusses implications for current management of global natural resources.

2. Model of a natural-resource-dependent economy

We consider a representative individual whose needs for two natural resources (h_1 and h_2) and a manufactured good (y) are described by the utility function

$$u(y, h_1, h_2) = y + \gamma \ln \left[\sum_{j=1,2} h_j^{1-\kappa} \right]^{\frac{1}{1-\kappa}} , \tag{1}$$

where $\gamma > 0$ is the representative household's weight of natural resources in the utility function and κ is the strength of demand-side interactions between the two resources. Its inverse $1/\kappa$ is the elasticity of substitution between the consumption of different natural resources. For different values of $\kappa > 0$, the degree of substitutability/complementarity between the natural resources varies. For $\kappa = 1$, sub-utility from consumption of resources would be the Cobb-Douglas function $\sqrt{h_1 \, h_1}$. For $\kappa \to 0$, the resources are perfect substitutes in consumption. For $\kappa \to \infty$, the resources are perfect complements. Overall, a higher value of κ implies a higher degree of complementarity.

The dynamics of the resource stocks (x_i), $i = 1, 2$ are described by the following differential equations

$$\dot{x}_j = f_j(x_j) - h_j \tag{2}$$

where the functions $f_j(\cdot)$ describe the intrinsic growth of the resource stocks, and h_j denotes the aggregate harvest of resource j. The equation of motion for resource j depends only on its own stock, i.e. we assume that the dynamics of different resources are independent. Although, of course, in reality there may exist ecological interactions between different natural resources, here we assume independence in order to focus on the effects of the durability of institutions and complementary needs. We specify the natural growth functions $f_j(\cdot)$ as logistic functions:

$$f_j(x_j) = r_j\, x_j \left(1 - \frac{x_j}{K_j}\right) \tag{3}$$

where r_j denotes the intrinsic growth rate and K_j the carrying capacity of resource j. With regard to the harvesting technology we assume Schaefer production functions (Gordon 1954, Schaefer 1957)

$$h_j = q_j\, x_j\, e_j \ , \tag{4}$$

where q_j is the productivity of harvesting the resource (in fisheries, it is often referred to as the 'catchability'), x_j is the stock of the resource, and e_j is the effort, i.e. labor, used for harvesting resource j.

The representative household inelastically supplies one unit of labor on a competitive market. Labor is divided across the two resource harvesting sectors and the manufactured-goods sector that produces the numeraire. In order to set up a general equilibrium model in a simple way, we assume that labor is the only factor input for the production of the manufactured good, and that production is through a constant-returns-to-scale technology, i.e. each unit of labor produces $\omega > 0$ units of output. Hence, aggregate output

of manufactured goods is

$$w = \omega \left(1 - \sum_{j=1,2} e_j \right) \tag{5}$$

and the (constant) competitive wage rate is equal to the marginal product of labor, ω. To ensure a positive consumption of the numeraire commodity, we assume $\omega > \gamma$ throughout the analysis (see footnote 2 below).

We assume that a central government is in place that has the aim to maximize the representative household's present value of utility. Furthermore, the economy's institutions are of limited permanence. With a positive probability of, the current institutions may be cease to exist at any given point in time by forces beyond the government's control. We assume that the societal discount rate δ captures both the representative household's impatience to consume and the limited permanence of the institutions. The government's objective function is thus given by

$$\int_0^\infty u(y, h_1, h_2)\, e^{-\delta\, t}\, dt \tag{6}$$

The government's optimization problem is to choose total harvest of the two resources, h_j and output of the manufactured good (w) such as to maximize (6) subject to (5), (4), and the equations of motion for the natural resources, equation (2).

When studying the resilience of the resource-dependent economy in the following, we consider exogenous disturbances that hit the economy. Such an exogenous disturbance is an unforeseen, one-time shock to the resource stocks. Considering unforeseen, one-time shocks implies that, after a disturbance, the economy will follow the path of resource use that is optimal given

the new initial stocks after the disturbance occurred.

3. Conditions for dynamically optimal resource use

To gain analytical insights into the dynamic properties of optimal resource use, we consider the current-value Hamiltonian

$$H = \gamma \frac{1}{1-\kappa} \ln \left[\sum_{j=1,2} h_j^{1-\kappa} \right] + \omega \left(1 - \sum_{j=1,2} \frac{h_j}{q_j \, x_j} \right) + \sum_{j=1,2} \mu_j \, [f_j(x_j) - h_j] \tag{7}$$

where μ_j are the shadow prices of the resource stocks. The first-order conditions for the government's optimization problem are

$$\gamma \, h_j^{-\kappa} \left[\sum_{j=1,2} h_j^{1-\kappa} \right]^{-1} = \frac{\omega}{q_j \, x_j} + \mu_j \qquad j = 1, 2 \tag{8}$$

$$\frac{\omega \, h_j}{q_j \, x_j^2} = \left[\delta - f_j'(x_j) \right] \mu_j - \dot{\mu}_j \qquad j = 1, 2 \tag{9}$$

together with the transversality conditions $e^{-\delta \, t} \, \mu_j \, x_j \xrightarrow{t \to \infty} 0$ and the condition that the initial resource stocks are given. For the following analysis, it is more convenient to use the shadow price of resource consumption, which we denote by π_i, instead of the shadow price of the resource stocks, μ_i. According to condition (8), this shadow price is

$$\pi_i = \frac{\omega}{q_j \, x_j} + \mu_j, \tag{10}$$

i.e., it is the sum of marginal harvesting cost and the shadow price μ_i of the resource stock, which is gives the marginal opportunity cost of harvesting. We obtain the consumption/harvest of resource i as a function of these shadow

prices from (8) after few steps of rearranging[2]

$$h_i = \gamma \frac{\pi_i^{-\frac{1}{\kappa}}}{\sum\limits_{j=1,2} \pi_j^{1-\frac{1}{\kappa}}} \tag{11}$$

Using (11) in (2) and (11) as well as (10) in (9), we obtain the following system of differential equations

$$\dot{x}_j = f_j(x_j) - \gamma \frac{\pi_i^{-\frac{1}{\kappa}}}{\sum\limits_{j=1,2} \pi_j^{1-\frac{1}{\kappa}}} \tag{12}$$

$$\dot{\pi}_j = \left[\delta - f_j'(x_j)\right] \left[\pi_j - \frac{\omega}{q_j\, x_j}\right] - \frac{\omega\, f_j(x_j)}{q_j\, x_j^2} \tag{13}$$

that governs the optimal dynamics of the resource-dependent economy together with the initial condition that the stocks at $t = 0$ are given and the transversality conditions, which become $e^{-\delta\, t}\, \pi_j\, x_j \xrightarrow{t\to\infty} 0$. The interaction between the two resources is captured by the harvesting term in equation (12). Equation (13), by contrast, depends only on the stock and shadow price of the resource $j = 1, 2$ itself.

The resilience of the resource-dependent economy is determined by the number and stability properties of the optimal steady states. A steady state is characterized by $\dot{x}_j = 0$ and $\dot{\pi}_j = 0$, $j = 1, 2$. With this, we obtain from

[2]Condition (11) shows that a positive consumption of the numeraire commodity is guaranteed, as $\sum_{j=1,2} \frac{\omega}{q_j\, x_j}\, h_j \leq \sum_{j=1,2} \pi_j\, h_j = \gamma < \omega$.

Equations (12) and (13) after few rearrangements

$$\pi_j = \pi_i \left[\frac{\gamma}{\pi_i \, f_i(x_i)} - 1 \right]^{\frac{\kappa}{\kappa-1}} \qquad \text{with } j, i \in \{1,2\} \text{ and } j \neq i \tag{14}$$

$$\pi_j = \frac{\omega}{q_j \, x_j} \left[1 + \frac{f_j(x_j)}{x_j \, \left[\delta - f_j'(x_j)\right]} \right] \qquad \text{with } j \in \{1,2\} \tag{15}$$

$$= \frac{\omega}{q_j \, x_j} \, \frac{\delta + r_j \, \frac{x_j}{K_j}}{\delta - r_j + 2 \, r_j \, \frac{x_j}{K_j}} \tag{16}$$

where we have used (3) to obtain the last expression. Again, it is the first of these equations that captures the interaction between the two resources. Note that the right hand side of condition (14) may be expressed as a function of the stock x_i of resource i only (as both π_i and $f_i(x_i)$ depend only on x_i), while the left hand side is a function of the stock x_j of resource $j \neq i$ only. Equating (15) and (14), we obtain two conditions that determine the steady-state stocks of the two resources $j = 1, 2$. Solving (16) for x_j yields

$$x_j(x_i) = \frac{K_j}{4} \left[\sqrt{\frac{8 \, \delta \, \omega}{r_j \, q_j \, K_j \, \pi_j} + \left[\frac{\delta - r_j}{r_j} - \frac{\omega}{K_j \, q_j \, \pi_j} \right]^2} - \frac{\delta - r_j}{r_j} + \frac{\omega}{K_j \, q_j \, \pi_j} \right] \tag{17}$$

with (from condition 14)

$$\pi_j = \frac{\omega}{q_i \, x_i} \, \frac{\delta + r_i \, \frac{x_i}{K_i}}{\delta - r_i + 2 \, r_i \, \frac{x_i}{K_i}} \left[\frac{\gamma \, q_i}{\omega \, r_i} \, \frac{\delta - r_i + 2 \, r_i \, \frac{x_i}{K_i}}{\left[\delta + r_i \, \frac{x_i}{K_i}\right] \left[1 - \frac{x_i}{K_i}\right]} - 1 \right]^{\frac{\kappa}{\kappa-1}} \tag{18}$$

Condition (17) with (18) gives the optimal steady-state stock of resource j as a function of the steady-state stock x_i of resource i, the $x_j(x_i)$-isocline.

4. Resilience of the resource-dependent economy

4.1. Analytical results

To be able to derive clear-cut analytical results, we assume in the following that both natural resources are governed by the same type of dynamics, i.e. the parameters of the biological growth functions and of the harvesting functions are the same: $r_1 = r_2 = r$, $K_1 = K_2 = 1$, and $q_1 = q_2 = q$. The major analytical advantage of symmetric resources is that a unique symmetric steady state exists.

Proposition 1. *With symmetric resources, and if* $2\,r\,\omega > \gamma\,q$*, one and only one symmetric steady state* $(x_1^\star, x_2^\star) = (x^\star, x^\star)$ *exists with*

$$x^\star = \frac{1}{2\,r}\left[r - \delta - \frac{\gamma\,q}{\omega} + \sqrt{(\delta + r)^2 + \left(\frac{\gamma\,q}{\omega}\right)^2}\right] > 0 \tag{19}$$

Proof. For a symmetric steady state $x^\star = x_1^\star = x_2^\star$, we have from condition (15) that $\pi_1^\star = \pi_2^\star$. With this, we conclude from condition (14) that the steady state is determined by the equation $2\,\pi\,f(x^\star) = \gamma$. The only positive solution to this equation is (19) (see appendix A.1) □

The resource stocks in the symmetric steady state do not depend on the degree of complementarity, κ, as in a symmetric steady state both resources are used in equal quantities. Of course, the resource stocks do depend on the societal discount rate, *delta*. It is easy to show that a larger discount rate leads to lower resource stocks. If the condition $2\,r\,\omega > \gamma\,q$ holds, however, the steady state stocks are positive even for $\delta \to \infty$.

Any optimal steady state, whether it is symmetric or not, is determined as the solution of the fixed point equation $x_1(x_2(x_1)) = x_1$, where both

$x_1(x_2)$ and $x_2(x_1)$ are determined by equation (17). In the following we study how the solutions to this fixed point equation depend on the degree of complementarity κ between the two resources. In the Cobb-Douglas-case $\kappa = 1$ the steady-state stocks of the two resources can be determined independent of each other: we see from equation (11) that in this case the steady-state condition (14) is replaced by the condition $r\,x_i\,(1 - x_i) = \gamma/(2\,\pi_i)$. In the following we study the case of weak and strong demand-side interactions, i.e. the cases where complementarity of the resources is weaker ($\kappa < 1$) or stronger ($\kappa > 1$) than in the Cobb-Douglas case. We will use the following lemma.

Lemma 1. *1. For weak demand-side interactions, i.e. if $\kappa < 1$, the $x_j(x_i)$-isoclines are strictly increasing over the whole range of x_i, $x_j'(x_i) > 0$ for $x_i \in [0, 1]$.*
2.a For strong demand-side interactions, i.e. if $\kappa > 1$, the $x_j(x_i)$-isoclines are strictly decreasing for a range of large x_i, i.e. a $\bar{x}_i \geq 0$ exists such that $x_j'(x_i) < 0$ for $x_i \in [\bar{x}_i, 1]$.
2.b For strong demand-side interactions, i.e. if $\kappa > 1$, and if $\delta > r\,\frac{\gamma\,q}{\gamma\,q - \omega\,r}$, the $x_j(x_i)$-isoclines have the properties that $x_j(0) = x_j(1) = 0$, with an interior maximum.

Proof. see appendix A.2. □

Proposition 1 shows that one (interior) steady state always exists. The questions are (i) whether this steady state is unique and (ii) if not, what are the stability domains of the different steady states. We consider the case of a low degree of complementarity ($\kappa < 1$) first.

Proposition 2. *For weak demand-side interactions, i.e. if $\kappa < 1$, the symmetric steady state $x^\star = x_j^\star = x_i^\star > 0$ given by (19) is the unique optimal steady state for any set of initial conditions (except for the zero-measure set of initial conditions where either $x_1(0) = 0$ or $x_2(0) = 0$ or both).*

Proof. For $\kappa < 1$ any steady state must be symmetric, as the following argument shows: Let $x^\star = x_1^\star = x_2^\star$ be a symmetric steady state. Since $x_j(x_i)$ is monotonically increasing (Lemma 1), it may be inverted, such that a steady state is determined by $x_2(x^\star) = x_1^{-1}(x^\star)$. For symmetric resources, we have $x_2(x) = x_1(x)$ for all x. Assume w.l.o.g. that $x_j'(x^\star) > 1$. Then, $x_i'(x^\star) = 1/x_j'(x^\star) < 1$. Thus, no asymmetric steady state is possible. Furthermore, only one symmetric steady state with $x^\star > 0$ exists (Proposition 1). □

A low degree of complementarity between resources thus ensures that the optimal development of the resource-dependent economy is also sustainable in the sense that in the steady state, the stocks of both resources are strictly positive and provide strictly positive resource rents. This may be completely different for a high degree of complementarity, $\kappa > 1$.

Proposition 3. *For strong demand-side interactions, i.e. if $\kappa > 1$ two asymmetric steady states $(x_1^{\star\star}, x_2^{\star\star})$ and $(x_2^{\star\star}, x_1^{\star\star})$ with $x_1^{\star\star} > 0$, $x_2^{\star\star} > 0$ and $x_1^{\star\star} \neq x_2^{\star\star}$ exist in addition to the symmetric steady state $(x_1^\star, x_2^\star) = (x^\star, x^\star)$ if the following conditions hold: (i) $\delta > r\,\frac{\gamma\,q}{\gamma\,q - \omega\,r}$ and (ii) $\kappa < -\frac{\pi'(x^\star)}{\pi(x^\star)}\,\frac{f(x^\star)}{f'(x^\star)}$, where $\pi(x^\star) = \frac{\omega}{q\,x^\star}\,\frac{\delta + r\,x^\star}{\delta - r + 2\,r\,x^\star}$ and $x^\star$ is given by (19).*

Proof. see appendix A.3 □

If several interior steady states exist, none of them is globally stable. This means that separate sets of initial conditions exist for which different steady

states are optimal. In the setting with two resources, considered here, these borders of these sets are initial conditions where the planner would optimally move to a third steady state.[3]

The first of the two conditions for several interior steady states to exist is that the discount rate is higher than a threshold value that is determined by the natural growth rate r of the resources, the weight γ of resources in utility and the technological parameters ω and q. It the discount rate gets higher than the threshold value, the symmetric steady state looses its global stability.

The second condition given in Proposition 3 is that the elasticity of substitution between the two resources exceeds a given threshold value (which, obviously, is smaller than one). If the elasticity of substitution would be below this threshold value, the symmetric steady state would be the only interior steady state, but it would be optimal only for an almost empty set of initial conditions. For most initial conditions, it would be optimal to deplete one of the resources, while the other one, which becomes useless if the complement is gone, ultimately reaches its natural equilibrium value. That is, the optimal steady states are $(0, 1)$ and $(1, 0)$.

4.2. Numerical examples

The numerical examples considered in this section are not meant to quantitatively resemble any specific resource-dependent economy. We have chosen intrinsic growth rates of 4% per year. Many natural resources such as ma-

[3]In models with one state variable, these borders are called Skiba-points after Skiba 1978. Here, the borders are curves rather than points.

rine fish stocks and different types of forests exhibit intrinsic growth rates in this order of magnitude. With regard to the technological parameters, we assume $\omega = 0.1$ and $q = 0.1$. The weight of resources in utility is $\gamma = 0.0667 = 0.667\,\omega$, which means that two thirds of time would have been spent harvesting resources.

These parameters fulfill the condition $2\,r\,\omega = 0.008 > \gamma\,q = 0.0667$. For $\delta > r\,\gamma\,q/(\gamma\,q - \omega\,r) = 0.1$, Condition (i) of Proposition 3 is fulfilled.

We varied the two other parameters of the model, i.e. the degree of complementarity κ and the discount rate δ in the calculations. We focus on the case of strong demand-side interactions, i.e. on a degree of complementarity $\kappa > 1$, as for weak demand-side interactions, $\kappa < 1$, the steady state is unique (Proposition 2).

Figure 1 shows the phase diagrams of the optimal resource dynamics given by Equations 12 and 13. In phase diagram (a) the degree of complementarity is $\kappa = 1.66$ and the societal discount rate is $\delta = 17\%$. For the phase diagram (b) we have used the same degree of complementarity as in (a), but the discount rate is only 9%. For the phase diagram (c) we have used the same discount rate as for diagram (a), but a higher degree of complementarity, namely $\kappa = 2.5$.

For $\delta = 17\% = 0.17 > r\,\gamma\,q/(\gamma\,q - \omega\,r) = 0.1$, Condition (i) of Proposition 3 is fulfilled. Condition (ii) is fulfilled for $\delta = 17\%$ if $\kappa < -\frac{\pi'(x^\star)}{\pi(x^\star)}\,\frac{f(x^\star)}{f'(x^\star)} = 1.88$. That means, for the phase diagram (a), both conditions of Proposition 3 are fulfilled. For phase diagram (b), condition (i) is not fulfilled, while for phase diagram (c), condition (ii) is not fulfilled. Accordingly, three interior steady states exist for the parameters used in diagram (a), while

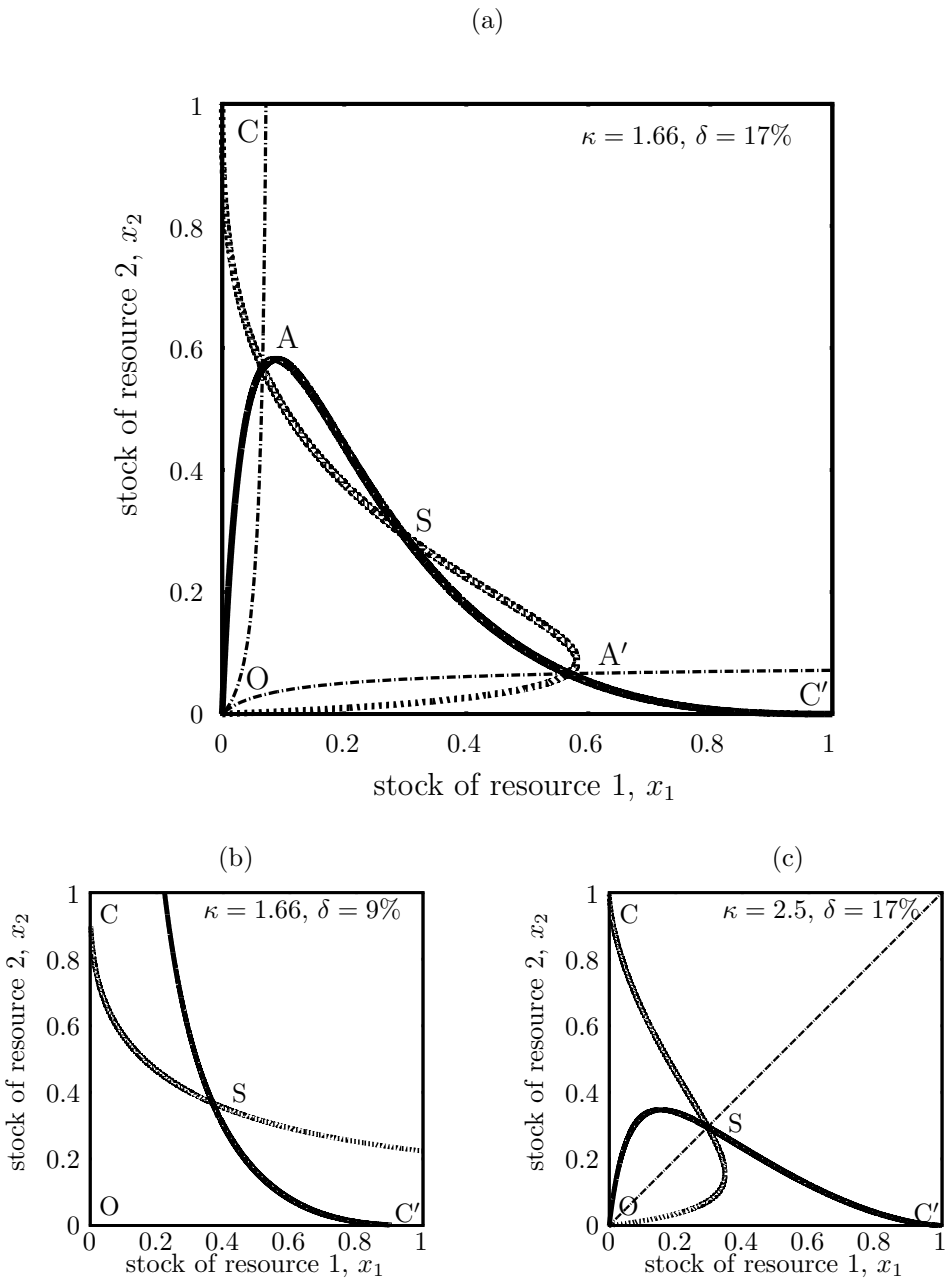

Figure 1: Phase diagrams for different degrees of complementarity κ of the resources and different societal discount rates δ.

exactly one interior steady state exists for the parameters used to construct diagrams (b) and (c). The stability properties of the steady states are determined by the eigenvalues of the Jacobian at the steady state (given in appendix A.4). For phase digram (a) in figure 1, the eigenvalues of the Jacobian are $(-0.0146, 1.90 \cdot 10^{-3}, 0.168, 0.185)$ for the steady states A and A$'$, and $(-0.0121, -1.84 \cdot 10^{-3}, 0.172, 0.182)$ for the steady state S. For phase diagram (b) the eigenvalues of steady state S are $(-0.0155, -6.65 \cdot 10^{-3}, 0.0966, 0.105)$, and for phase diagram (c) the eigenvalues of steady state S are $(-0.0121, 3.76 \cdot 10^{-3}, 0.166, 0.182)$

Figure 1 (a) shows the most interesting dynamics, as three interior steady states exist. The two asymmetric steady states A and A$'$ are saddle-point stable. The dotted lines depict the saddle-path that would lead to these steady states. For all initial states of the resource stocks, the optimal paths would tend towards the symmetric steady state S. For initial states to the east of the saddle path in the north-west, the optimal steady state is C, while for initial states below the saddle path in the south-east, the optimal steady state is C$'$.

The symmetric steady state in Figure 1 (c) is globally stable. In Figure 1 (b), the complementarity of resources and the rate of discount are so high that the collapse of the natural-resource-dependent economy is optimal for almost any set of initial conditions. The only exception is the case where the economy initially is on the saddle-path to the symmetric steady state, i.e. where $x_1(0) = x_2(0)$ holds exactly.

The dependency of the equilibria and their stability domains on (a) the discount rate and (b) the degree of complementarity of the two resources is

illustrated in the bifurcation diagrams shown in Figure 2. In the graphs, solid lines depict an (almost) globally stable equilibrium, dotted lines (almost) unstable equilibria , and dashed lines locally stable equilibria.

5. Conclusion

We have analyzed how characteristics of consumer preferences affect the (limited) resilience of natural-resource dependent economies. Thereby, we have focused on the complementarity of resources in the satisfaction of human needs, and the societal discount rate. We have derived conditions on the degree of complementarity and on the discount rate for which the optimal dynamics of resource use features multiple steady states and path-dependence. We have shown that the resilience of natural-resource-dependent economies decreases with the degree of complementarity between resources in consumer preferences.

Pointing to the strength of economic interactions between different resources for an explanation of the systems stability properties opens potentially fruitful perspectives for the discussion of stability of economic systems. Generally, we hypothesize that the stability of economic systems decreases with the strength of demand-side interactions.

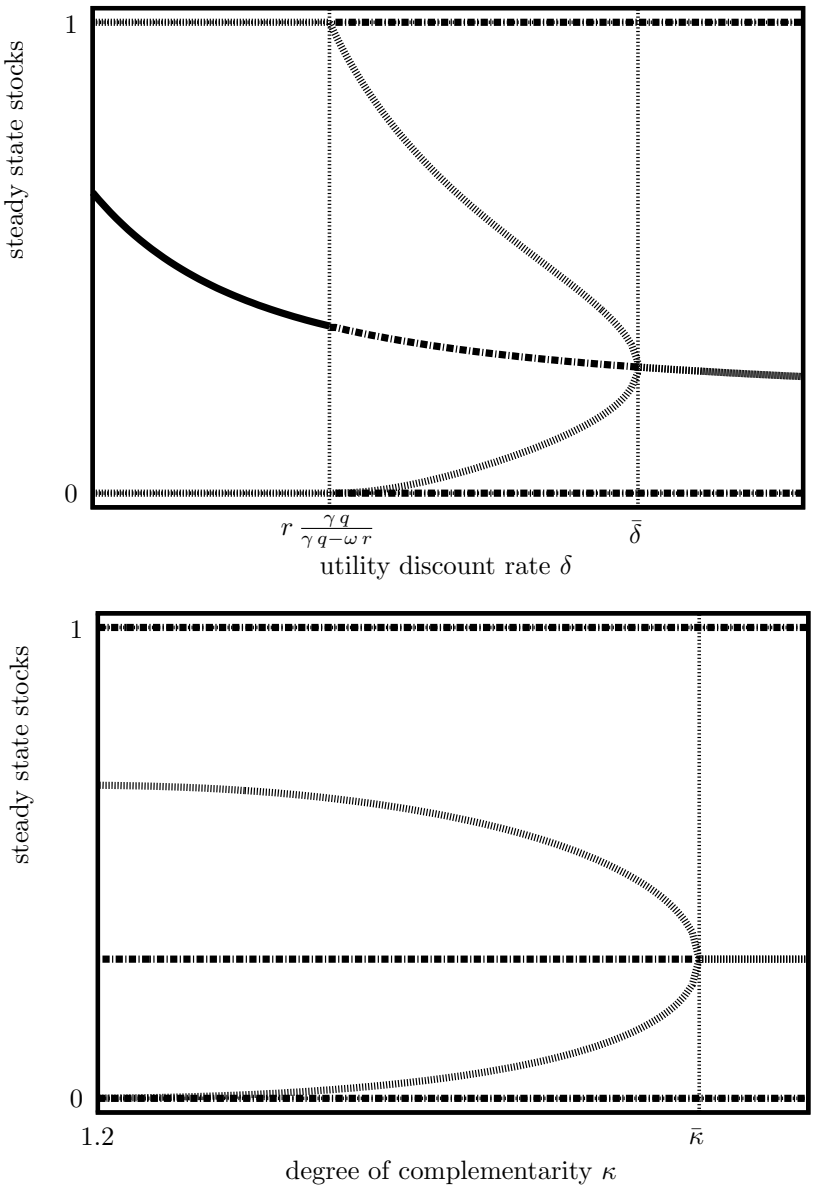

Figure 2: Bifurcation diagrams: steady states as function of discount rate (top) and degree of complementarity (bottom). Solid lines depict an (almost) globally stable equilibrium, dotted lines (almost) unstable equilibria , and dashed lines locally stable equilibria.

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A.1 (Symmetric steady state)**.** *Using* (15) *and* (3) *in the condition* $2\,\pi_j\,f(x^\star) = \gamma$ *for a symmetric steady state yields*

$$2\,\frac{\omega}{q}\,\frac{\delta + r\,x^\star}{\delta - r + 2\,r\,x^\star}\,r\,(1 - x^\star) = \gamma \tag{A.1}$$

$$(\delta + r\,x^\star)\,r\,(1 - x^\star) = \frac{\gamma\,q}{2\,\omega}\,(\delta - r + 2\,r\,x^\star) \tag{A.2}$$

$$\delta\,r + (r - \delta)\,r\,x^\star - (r\,x^\star)^2 = \frac{\gamma\,q}{2\,\omega}\,(\delta - r + 2\,r\,x^\star) \tag{A.3}$$

$$(r\,x^\star)^2 + \left(\delta - r + 2\,\frac{\gamma\,q}{2\,\omega}\right)\,r\,x^\star = r\,\delta - \frac{\gamma\,q}{2\,\omega}\,(\delta - r) \tag{A.4}$$

$$\left(2\,r\,x^\star + \delta - r + \frac{\gamma\,q}{\omega}\right)^2 = (\delta + r)^2 + \left(\frac{\gamma\,q}{\omega}\right)^2 \tag{A.5}$$

Solving for $x^\star$ *yields*

$$x^\star = \frac{1}{2\,r}\left[r - \delta - \frac{\gamma\,q}{\omega} \pm \sqrt{(\delta + r)^2 + \left(\frac{\gamma\,q}{\omega}\right)^2}\right] \tag{A.6}$$

A.2 (Proof of Lemma 1)**.** *Ad 1. Note first that any real-valued solution to* (14) *requires that* π_j, $j = 1, 2$ *is positive. Using* (3) *in* (15)*, this leads to the conclusion that for any steady state* $x_j > (r - \delta)/(2\,r)$*. Differentiating* (15) *with respect to* x_j *and using* (3) *yields*

$$\frac{d\pi_j}{dx_j} = -\,\frac{\omega}{q}\,\frac{2\,[r\,x_j]^2 + [\delta]\,[\delta - r + 4\,r\,x_j]}{x_j^2\,[\delta - r + 2\,r\,x_j]^2} \tag{A.7}$$

This implies that the shadow price of harvest of resource j *is monotonically decreasing with the stock* x_j*, as* $x_j > (r - \delta)/(2\,r)$*. In a similar way, it is easily verified that* $d^2\pi_j/dx_j^2 > 0$ *for the relevant range of stocks,* $x_j > (r - \delta)/(2\,r)$*. Total differentiation of* (14) *leads to*

$$\frac{d\pi_j}{dx_j}\,\frac{dx_j}{dx_i} = \frac{d\pi_i}{dx_i}\left[\frac{\gamma}{\pi_i\,f(x_i)} - 1\right]^{\frac{\kappa}{\kappa-1}} - \frac{\gamma\,\left[\frac{\gamma}{\pi_i\,f(x_i)} - 1\right]^{\frac{1}{\kappa-1}}}{\frac{\kappa-1}{\kappa}\,f(x_i)}\,\left[\frac{1}{\pi_i}\,\frac{d\pi_i}{dx_i} + \frac{f'(x_i)}{f(x_i)}\right] \tag{A.8}$$

The first term on the right hand side is negative, $d\pi_i/dx_i < 0$, and as by equation (14) $\gamma/(\pi_i\, f(x_i) > 1$. The last factor (in brackets) of the second term on the right hand side is negative, too:

$$\frac{1}{\pi_i}\,\frac{d\pi_i}{dx_i} + \frac{f'(x_i)}{f(x_i)} = -\frac{2\,[r\,x_i]^2 + \delta\,[\delta - r + 4\,r\,x_i]}{[\delta + r\,x_i]\,[\delta - r + 2\,r\,x_i]} + \frac{1 - 2\,x_i}{x_i\,[1 - x_i]}$$
$$= -\frac{\delta\,[\delta + 2\,r\,x_i] + r^2\,[1 - 2\,x_i\,[1 - x_i]]}{[1 - x_i]\,[\delta + r\,x_i]\,[\delta - r + 2\,r\,x_i]} < 0 \quad \text{(A.9)}$$

For $\kappa > 1$, the first factor of the last term in Equation (A.8) is positive. In conclusion, we have $dx_j/dx_i > 0$, as $d\pi_j/dx_j < 0$.

Ad 2. If $x_i = 1$, we have $\pi_i|_{x_i=1} = \omega/q_i > 0$, i.e. the shadow price of resource use equals the marginal harvesting cost, as the shadow price of the stock is zero. Using this in (14), we have (with $\kappa > 1$) $\pi_j \xrightarrow{x_i \to 1} \infty$, i.e. harvest (equal to natural growth) of stock i is zero in the steady state (only) if the shadow price of the complementary resource j is prohibitively high. This, in turn, implies $x_j(1) = 0$ (by equation 17). As $x_j(x_i) \geq 0$ and as $x_j(x_i)$ is continuous in $x_i < 1$, it follows that $x_j'(x_i) < 0$ in a neighborhood of 1.

Ad 3. That $x_j(1) = 0$ has been shown in the last paragraph. Under the condition given in the lemma, we find

$$\lim_{x_i \to 0} \left[\frac{\gamma}{\pi_i\, f(x_i)} - 1\right]^{\frac{\kappa}{\kappa-1}} = \left[\frac{\gamma}{\frac{\omega}{q}\,\frac{\delta}{\delta - r}\,r} - 1\right]^{\frac{\kappa}{\kappa-1}} = \left[\frac{\gamma\,q}{\omega\,r}\,\left[1 - \frac{r}{\delta}\right] - 1\right]^{\frac{\kappa}{\kappa-1}} \quad \text{(A.10)}$$

which is real and greater than zero for

$$\frac{\gamma\,q}{\omega\,r}\,\left[1 - \frac{r}{\delta}\right] > 1 \quad \text{(A.11)}$$
$$\frac{r}{\delta} < 1 - \frac{\omega\,r}{\gamma\,q} = \frac{\gamma\,q - \omega\,r}{\gamma\,q} \quad \text{(A.12)}$$
$$\delta > r\,\frac{\gamma\,q}{\gamma\,q - \omega\,r} \quad \text{(A.13)}$$

Since, by condition (15)*,* $\pi_i \xrightarrow{x_i \to 0} \infty$*, it follows from* (14) *that* $\pi_j \xrightarrow{x_i \to 0} \infty$*. Using this in* (17) *shows* $x_j(0) = 0$*. Since* $x_j(x_i)$ *must be positive for some* $x_i \in [0, 1]$*, this function must assume an interior maximum in the domain* $[0, 1]$*.*

A.3 (Proof of proposition 3)**.** *The symmetric steady state exists in any case (proposition 1). We now show that under the conditions given in the proposition, two more interior steady states exist. For this sake we show that the equation* $x_2(x_1(x_2)) = x_2^{\star\star}$ *must (also) have a solution* $x_2^{\star\star} > x^\star$*. From lemma 1 (2.b) we have that* $x_2(x_1(1)) = x_2(0) = 0 < 1$*. In the symmetric steady state, we have* $x_2(x^\star) = x_1(x^\star) = x^\star$*,* $\pi_1 = \pi_2 = \pi(x^\star)$*,* $d\pi_1/dx_1 = d\pi_2/dx_2 = \pi'(x^\star)$ *and* $\gamma/(\pi(x^\star)\, f(x^\star)) - 1 = 1$ *(see appendix A.2). Under the condition* $\kappa < -\frac{\pi_1'}{\pi_1}\frac{f(x^\star)}{f'(x^\star)}$*, we have* $x_2'(x^\star) < -1 < 1/x_1'(x^\star)$*, as equation* (A.8) *simplifies to*

$$\frac{dx_j}{dx_i} = 1 - \frac{2\,\kappa}{\kappa - 1}\left[1 + \frac{\pi_1}{\frac{d\pi_1}{dx_i}}\frac{f'(x^\star)}{f(x^\star)}\right] \tag{A.14}$$

$$< 1 - \frac{2\,\kappa}{\kappa - 1}\left[1 - \frac{1}{\kappa}\right] = 1 - 2 = -1 \tag{A.15}$$

Hence, $x_2(x^\star - \epsilon) > x_1(x^\star - \epsilon)$ *for some small* $\epsilon > 0$*. Since both* $x_2(\cdot)$ *and* $x_1(\cdot)$ *are continuous, the equation* $x_2(x_1(x_2)) = x_2^{\star\star}$ *must have a solution* $x_2^{\star\star} > x^\star$*.*

A.4 (Jacobian matrix at steady state)**.**

$$\begin{pmatrix} f_1'(x_1) & 0 & \gamma\,\frac{\pi_1^{-\frac{1}{\kappa}-1}\,\pi_2^{1-\frac{1}{\kappa}}+\pi_1^{-2\,\frac{1}{\kappa}}}{\kappa\left(\pi_1^{1-\frac{1}{\kappa}}+\pi_2^{1-\frac{1}{\kappa}}\right)^2} & \gamma\,\frac{(1-\frac{1}{\kappa})\,\pi_1^{-\frac{1}{\kappa}}\,\pi_2^{-\frac{1}{\kappa}}}{\left(\pi_1^{1-\frac{1}{\kappa}}+\pi_2^{1-\frac{1}{\kappa}}\right)^2} \\ 0 & f_2'(x_2) & \gamma\,\frac{(1-\frac{1}{\kappa})\,\pi_1^{-\frac{1}{\kappa}}\,\pi_2^{-\frac{1}{\kappa}}}{\left(\pi_1^{1-\frac{1}{\kappa}}+\pi_2^{1-\frac{1}{\kappa}}\right)^2} & \gamma\,\frac{\frac{1}{\kappa}\,\pi_2^{-\frac{1}{\kappa}-1}\,\pi_1^{1-\frac{1}{\kappa}}+\pi_2^{-2\,\frac{1}{\kappa}}}{\left(\pi_1^{1-\frac{1}{\kappa}}+\pi_2^{1-\frac{1}{\kappa}}\right)^2} \\ \frac{\partial\dot{\pi}_1}{\partial x_1} & 0 & \delta - f_1'(x_1) & 0 \\ 0 & \frac{\partial\dot{\pi}_2}{\partial x_2} & 0 & \delta - f_2'(x_2) \end{pmatrix} \tag{A.16}$$